Seeing through the string landscape—a string hunter's companion in particle physics and cosmology

Review

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Dieter Lüst

Arnold Sommerfeld Center for Theoretical Physics,
LMU-München, Theresienstr. 37, 80333 München, Germany
Max-Planck-Institut für Physik,
Föhringer Ring 6, 80805 München, Germany

E-mail: dieter.luest@lmu.de

Abstract: In this article we will overview several aspects of the string landscape, namely intersecting D-brane models and their statistics, possible model independent LHC signatures of intersecting brane models, flux compactification, moduli stabilization in type II compactifications, domain wall solutions and brane inflation.

Contents

1 Introduction

Particle physics and cosmology are entering an unprecedentedly exciting epoch. We are about to be confronted with new experimental data which will provide entirely new information about the structure of matter, the fundamental interactions at short distances as well as about the history of the early universe. Most notably the LHC experiment is expected to discover (or exclude) the Higgs particle and hence will test our picture about the origin of mass in the Standard Model (SM). In addition, the LHC might discover completely new particles in the TeV region as predicted e.g. by supersymmetry. This would be a clear signal for physics beyond the SM. In astroparticle physics, exciting new experiments such as the Planck satellite and others will provide further data. This will allow to pin down the parameters of cosmic inflation, dark energy and dark matter with a much higher precision than before. Moreover laser interferometers (LIGO and LISA) may for the first time discover gravitational waves with possibly far reaching consequences for our understanding of the early universe. As is clear by now, progress in particle physics and cosmology go hand in hand. For example new particles discovered at LHC may serve as dark matter candidates. On the other hand, to understand cosmic inflation and dark energy at a microscopic level requires concrete underlying particle physics models for the physics of the early universe.

In order to describe the physics at very high energies and during the very early universe, new theoretical concepts are necessary which go beyond the SM of particle physics. Among various attempts in this direction, string theory is perhaps the most successful and also the most ambitious approach since besides the gauge interactions it includes also the gravitational force at the quantum level (for some textbooks on string theory see [1–5]). Recently there has been achieved substantial progress in connecting string theory with particle physics and cosmology. In addition, in spite of many open problems, it has become clear that string compactifications allow for a huge number of possible ground states, nowadays referred to as the landscape of string theory vacua.

The apparent existence of a string landscape is a double-edged sword. Several of these string vacua possess attractive phenomenological properties in that they come very close to the SM of particle physics or to realistic models of cosmic inflation. Also the problem

of dark energy was recently addressed in string theory in an interesting way. On the other hand, it was hoped for a long time that the fundamental theory of nature would allow only for a single vacuum which would explain all physical phenomena and make unique predictions for future observations. As it stands at present, this hope may have been too naive. In fact, the existence of a large number of solutions of a physical theory is nothing specific to string theory. Every sufficiently complex gauge theory allows for a large number of at least meta-stable vacua whose phenomenological properties vary considerably.

Still, the emergence of the string landscape and the attempts for its interpretation may mark a shift in paradigm of how to treat the problem of unification and also of uniqueness in a fundamental theory. As a totally new aspect that developed recently, the search for realistic string vacua has lead to the application of statistical methods. In a similar spirit it is argued (and heavily debated) that the problem of explaining the cosmological constant (and perhaps even other constants in nature) can be solved by some anthropic interpretation of the string landscape.

The scope of the article is to explain recent developments in string theory with special emphasis on string compactifications from ten to four space-time dimensions and the associated landscape of string vacua. We are planning to discuss the following two main aspects of the string landscape:

- How string theory connects to the real world in particle physics and cosmology. Particular emphasis will be put on how to derive the SM from string compactifications and how to get viable models for cosmic inflation and for the description of dark energy. We will also discuss possible string signatures at the LHC collider experiment.

- How we describe and how we deal with the huge landscape in string theory. We will explain that statistical methods are useful for the search of the SM from string theory. In addition, we will discuss some general aspects of the landscape like transitions between different vacua and constraints from black hole decays.

One possible approach to string theory is the *top-down* approach, which starts from the unification of gravity and gauge interactions at very high energies, and then tries to deduce all low energy observables from investigating the mathematical structures of the theory. Although we do not yet know the typical string scale M_{string}, where the unification of gravity and gauge interaction takes places, one often assumes, at least from a conservative point of view, that this happens at the Planck scale of about $M_{\text{Planck}} \simeq 10^{19}$ GeV. However no direct experiments will guide us through the physics at such high energies, a fact which makes the top-down approach very troublesome. But we like to emphasize already at this point that the string scale M_{string} a free parameter, which a priori can take any value, especially in type II orientifold models, where the SM lives on lower dimensional branes. In fact, as we will discuss later, intersecting brane models with all low string scale allow for model-independent predictions at collider experiments, most notably at the LHC, which are generic for a large class of models inside the string landscape.

Let us be more precise what we actually mean by the string landscape. It is defined to be the space of all possible solutions of the string equations of motion. In ten space-time di-

mensions, there exist just five different formulations of string theory (two heterotic strings, type I type IIA and type IIB superstrings). Exploring several kind of duality symmetries, it is conjectured that all these string theories can be unified into M-theory, where also 11-dimensional supergravity is included. However the number of lower-dimensional string solutions, i.e. lower dimensional string ground states, which are obtained after compactification, is enormous. This fact became clear already in 1986 constructing heterotic strings in four dimensions [6–8], and within the covariant lattice construction [7], the number of possible four-dimensional string ground states was estimated to be of order 10^{1500}. More recently, the number of discrete flux vacua of an effective supergravity potential for type II compactifications on a generic Calabi-Yau manifold was shown to be of order 10^{500} [9, 10]. Taken seriously, this vast landscape of distinct string vacua really implies a big question mark concerning the predictivity of string theory, since each point in the landscape essentially corresponds to a different universe with different particle physics and cosmological properties. To deal with such a huge number of possibilities, certain strategies are required in order to proceed within the top-down approach. One possible and legitimate approach is given by the investigation of the statistical properties of the string landscape. i.e. one has to determine by statistical methods what is the fraction of string vacua with good phenomenological properties. Possible statistical correlations resp. anti-correlations would be especially worth to be discovered, like e.g. between the number of families and the rank of the low-energy gauge groups, because they could provide a step towards verifying or resp. falsifying string theory. Eventually, the statistical approach is likely to be merged with the anthropic principle [11] (see also [12]). Concerning the evolution of the universe (see e.g. [13]), the anthropic principle essentially requires a multiverse with a huge number of bubbles, with each being filled by one of the vacua of the landscape. The population of all possible bubbles in the universe is possible in the context of eternal inflation, where transitions between different bubbles due to quantum tunneling processes are going to happen.

Complementary to the top-down efforts, the *bottom-up* approach is very important for connecting string theory with the real world. Here one tries to build consistent string models which contain as many SM features as possible. First one tries to build string models that contain as massless states the particles of the SM, gauge bosons and three families of quarks and leptons. Next, one has to derive the low-energy effective action of the massless fields, in order to compute their couplings, like gauge couplings and Yukawa couplings, which eventually can be compared with the experimentally known values. Here another problem has to be solved, namely the problem of moduli stabilization. String compactifications generically contain several massless moduli fields with flat potential, which correspond to geometrical or other parameters of the internal space. These have to be fixed, since the low-energy couplings of the massless fields are functions of the moduli. In order to make predictions one has to know the values of the moduli. In addition, massless moduli would over-close the universe and also cause unobserved new forces [14].

The bottom-up approach is especially useful in case some model independent and possibly testable properties rise just from the very fact that the SM has to be consistently embedded into string theory. As we will discuss this happens for type II orientifold compactifications with a low string scale around the TeV scale. In fact, the occurrence of

Regge excitations of SM fields is independent from the details of the internal geometry of the compactification. If light enough, these Regge states can be possibly measured at the LHC by scattering processes of quarks and gluons. The corresponding tree level string cross sections are independent from in the internal geometry and hence independent from the particular location of the model in the string landscape. This observation nullifies in some sense the string landscape problem at the LHC.

As already mentioned, another window into new physics beyond the SM comes from astrophysics and cosmology. Beautiful experiments, most notably COBE and WMAP, provided a precise image of cosmic microwave background (CMB) radiation including its small density variations. In this way the inflationary scenario of the early universe is now established as the standard model for cosmology. In addition, we know from the astrophysical measurements that our universe is spatially flat. Its energy density is dominated to about 74% by a dark energy component, which behaves very similarly to a positive cosmological constant. The explanation of this mysterious dark energy is one of the biggest challenges for astroparticle physics, and hence also for string theory. The remaining 26% of the energy density is split into so far directly undiscovered dark matter particles (WIMPS), which account for 22% of the total energy density, and into a left-over 4% component of visible SM matter fields. Many properties of the dark matter fields are still unknown, although one very promising candidate for dark matter is the lightest supersymmetric particle (LSP) in the MSSM. So, strings also should be helpful to identify the nature of dark matter. Hence the goal will be to use the data from the CMB, from dark matter and from other astrophysical experiments to find or to probe the fundamental theory of strings in the early universe. This will put further constraints on the allowed points in the string landscape, often complementary to the particle physics constraints mentioned before.

In summary, successful string model building must take into account all these phenomenological boundary conditions coming from the SM, from particle physics beyond the SM and also from cosmology. The top-down constructions which start from the geometry of the compactification space must go hand in hand with the bottom-up approach, where one is guided by the phenomenological data. In this way, a (not necessarily one-to-one) map between geometrical and topological properties of the compactifications spaces and the particle physics observables will be provided. This dictionary between geometry/topology and particle physics/cosmology is one of the most interesting aspects of string theory, and will be demonstrated in this paper by several examples. Of course, it still has to be seen in the future if a string compactification can be found that matched combined constraints from particle physics as well as from early time cosmology.

2 Type II intersecting brane models and their statistics

2.1 Overview over different classes of orientifold models

In this section we will review how realistic string compactifications can be built from intersecting D-brane orientifolds. They constitute a large class of models in the string landscape. The orientifold region in the landscape is complementary, and for type I strings often dual, to heterotic string compactifications, which will be left out here. Specifically

consider type II orientifold compactifications to four-dimensions on six-dimensional manifolds $\mathcal{M}_6$, which were first discussed in [15–19] (for some reviews see [20–24]). In order to incorporate non-Abelian gauge interactions and to obtain massless fermions in non-trivial gauge representations, one has to introduce D-branes in type II superstrings. Specifically there exist three classes of four-dimensional models:

(i) *Type I compactifications with D9/D5 branes:*

This class of IIB models contain different stacks of D9-branes, which wrap the entire space $\mathcal{M}_6$, and which also possess open string, magnetic, Abelian gauge fields F_{ab} on their world volumes (magnetized branes). In other words, F_{ab} corresponds to open string vector bundles, and this class of models is string dual to heterotic string compactifications. For reasoning of Ramond tadpole cancellation, one also needs an orientifold 9-plane (O9-plane). In addition one can also include D5-branes and corresponding O5-planes. In the heterotic dual description the D5/O5 open strings correspond to the non-perturbative sector of the theory. Since the open string gauge fields F_{ab} induce mixed boundary conditions on the D-branes, the internal compact space can be regarded as a non-commutative space.

(ii) *Type IIB compactifications with D7/D3 branes:*

Here we are dealing with different stacks of D7-branes, which wrap different internal 4-cycles, which intersect each other. The D7-branes can also carry non-vanishing open string gauge flux F_{ab}. In addition, one can also allow for D3-branes, which are located at different point of $\mathcal{M}_6$. In order to cancel all Ramond tadpoles one needs in general O3- and O7-planes. Recently interesting GUT SU(5) embeddings with D7-branes wrapped on Calabi-Yau cycles and U(1)$_Y$ background fluxes were constructed Blumenhagen:2008at, which can be also formulated in F-theory [26, 27].

(iii) *Type IIA compactifications with D6 branes:*

This class of models contains intersecting D6-branes, which are wrapped around 3-cycles of $\mathcal{M}_6$. Now, orientifold O6-planes are needed for Ramond tadpole cancellation. One can show that the cancellation of the RR tadpoles implies absence of the non-Abelian anomalies in the effective 4D field theory. However there can be still anomalous U(1) gauge symmetries in the effective 4D field theory. These anomalies will be canceled by a Green-Schwarz mechanism involving Ramond (pseudo)scalar field. As a result of these interactions the corresponding U(1) gauge boson will become massive. Note that even an anomaly free U(1) can become massive. The massive U(1) always remains as a global symmetry. For SM engineering, we always have to require that the linear combination of U(1)'s that corresponds to U(1)$_Y$ is anomaly free and massless.

2.2 Intersecting D6-brane orientifolds

D-brane models of these three different classes generically can be mapped onto each other by T-duality, resp. IIA/IIB mirror symmetry including open strings and D-branes, and

Sector	Rep.	Intersection number I
$a'\,a$	A_a	$\frac{1}{2}\left(\pi_a' \circ \pi_a + \pi_{O6} \circ \pi_a\right)$
$a'\,a$	S_a	$\frac{1}{2}\left(\pi_a' \circ \pi_a - \pi_{O6} \circ \pi_a\right)$
$a\,b$	$(\overline{N}_a, N_b)$	$\pi_a \circ \pi_b$
$a'\,b$	(N_a, N_b)	$\pi_a' \circ \pi_b$

Table 1. Number of representations in each intersection sector in terms of the intersection numbers.

therefore are essentially on equal footing. Hence, in the following we will concentrate on IIA intersecting D6-brane models (class (iii)).

Let us therefore just summarize the main aspects of the intersecting D6-brane models.

- We assume that six spatial directions are described by a compact space $\mathcal{M}_6$. In addition, a consistent orientifold projection is performed. This yields O6-planes and in general changes the geometry. The bulk space-time supersymmetry is reduced to $\mathcal{N} = 1$ by the orientifold projection. To be more specific we will consider a type IIA orientifold background of the form

$$\mathcal{M}^{10} = (\mathbb{R}^{3,1} \times \mathcal{M}_6)/(\Omega\overline{\sigma}), \quad \Omega : \text{world sheet parity}. \tag{2.1}$$

Here $\mathcal{M}_6$ is a Calabi-Yau 3-fold with a symmetry under $\overline{\sigma}$, the complex conjugation

$$\overline{\sigma} : z_i \mapsto \bar{z}_i, \quad i = 1, \ldots, 3, \tag{2.2}$$

in local coordinates $z_i = x^i + iy^i$. It is combined with the world sheet parity Ω to form the orientifold projection $\Omega\overline{\sigma}$. This operation is actually a symmetry of the type IIA string on $\mathcal{M}_6$. Orientifold 6-planes are defined as the fixed locus

$$\mathbb{R}^{3,1} \times \text{Fix}(\overline{\sigma}) = \mathbb{R}^{3,1} \times \pi_{O6},$$

where $\text{Fix}(\overline{\sigma})$ is a supersymmetric (sLag) 3-cycle on $\mathcal{M}^6$, denoted by π_{O6}. It is special Lagrangian (sLag) and calibrated with respect to the real part of the holomorphic 3-form Ω_3.

Next we introduce D6-branes with world-volume

$$\mathbb{R}^{3,1} \times \pi_a,$$

i.e. they are wrapped around the supersymmetric (sLag) 3-cycles π_a and their $\Omega\overline{\sigma}$ images π_a' of $\mathcal{M}_6$, which intersect in $\mathcal{M}_6$. Since the D-branes will be wrapped around compact cycles of the internal space, multiple intersections will now be possible. The chiral massless spectrum indeed is completely fixed by the topological intersection numbers I of the 3-cycles of the configuration.

- Since the Ramond charges of the space-time filling D-branes cannot 'escape' to infinite, the internal Ramond charges on compact space must cancel (Gauss law). This is the issue of Ramond tadpole cancellation which give some strong restrictions on the allowed D-brane configurations. Specifically, the Ramond tadpole conditions follow from the equations of motion for the gauge field C_7:

$$\frac{1}{\kappa^2} d \star dC_7 = \mu_6 \sum_a N_a \, \delta(\pi_a) + \mu_6 \sum_a N_a \, \delta(\pi_a') + \mu_6 Q_6 \, \delta(\pi_{\mathrm{O6}}), \qquad (2.3)$$

where $\delta(\pi_a)$ denotes the Poincaré dual form of π_a, $\mu_p = 2\pi(4\pi^2\alpha')^{-(p+1)/2}$, and $2\kappa^2 = \mu_7^{-1}$. Upon integrating over $\mathcal{M}_6$ one obtains the RR-tadpole cancellation as equation in homology:

$$\sum_a N_a \, (\pi_a + \pi_a') - 4\pi_{\mathrm{O6}} = 0. \qquad (2.4)$$

In principle it involves as many linear relations as there are independent generators in $H_3(\mathcal{M}_6, R)$. But, of course, the action of $\overline{\sigma}$ on $\mathcal{M}_6$ also induces an action $[\overline{\sigma}]$ on the homology and cohomology. In particular, $[\overline{\sigma}]$ swaps $H^{2,1}$ and $H^{1,2}$, and the number of conditions is halved.

- Next, there is the requirement of cancellation of the internal D-brane tensions, i.e the forces between the D-branes must be balanced. In terms of string amplitudes, it means that all NS tadpoles must vanish, namely all NS tadpoles of the closed string moduli fields and also of the dilaton field. Absence of these tadpoles means that the potential of those fields is minimized. The disc level tension can be determined by integrating the Dirac-Born-Infeld effective action. It is proportional to the volume of the D-branes and the O-plane, so that the disc level scalar potential reads

$$\mathcal{V} = T_6 \, \frac{e^{-\phi_4}}{\sqrt{\mathrm{Vol}(\mathcal{M}_6)}} \left(\sum_a N_a \left(\mathrm{Vol}(\mathrm{D6}_a) + \mathrm{Vol}(\mathrm{D6}_a') \right) - 4\mathrm{Vol}(\mathrm{O6}) \right)$$

$$= T_6 \, e^{-\phi_4} \left(\sum_a N_a \left| \int_{\pi_a} \widehat{\Omega}_3 \right| + \sum_a N_a \left| \int_{\pi_a'} \widehat{\Omega}_3 \right| - 4 \left| \int_{\pi_{\mathrm{O6}}} \widehat{\Omega}_3 \right| \right). \qquad (2.5)$$

The potential is easily seen to be positive semi-definite and its minimization imposes conditions on some of the moduli, freezing them to fixed values. Whenever the potential is positive, supersymmetry is broken and a classical vacuum energy generated by the net brane tension. It is easily demonstrated that the vanishing of $\mathcal{V}$ requires all the cycles wrapped by the D6-branes to be calibrated with respect to the same 3-form as are the O6-planes.

- One can show that the cancellation of the RR tadpoles implies absence of the non-Abelian anomalies in the effective 4D field theory. However there can be still anomalous U(1) gauge symmetries in the effective 4D field theory. These anomalies will be canceled by a Green-Schwarz mechanism involving Ramond (pseudo)scalar field. As a result of these interactions the corresponding U(1) gauge boson will become

massive. Considering the relevant triangle diagrams the condition for an anomaly free U(1)$_a$ is:

$$N_a(\pi_a - \pi'_a) \circ \pi_b = 0 \,. \tag{2.6}$$

Note that even an anomaly free U(1) can become massive. The massive U(1) always remains as a global symmetry. For SM engineering, we always have to require that the linear combination of U(1)'s that corresponds to U(1)$_Y$ is anomaly free and massless.

- Besides the local triangle anomalies, field theoretical models can be plagued by global SU(2) gauge anomalies. In orientifold models this requirement can be deduced from a K-theory analysis. In the case of our models, this condition requires an even amount of chiral matter from Sp(2) probe branes. In this case we obtain the following condition for a model with k stack of branes:

$$\sum_{a=1}^{k} N_a \pi_a \circ \pi_p \equiv 0 \mod 2 \,. \tag{2.7}$$

This equation should hold for any probe brane p invariant under the orientifold map.

2.3 Getting the Standard Model

Now we will discuss how the (supersymmetric) Standard Model (SM) can be obtained from intersecting brane orientifolds. In general there will be two different brane sectors, namely one local D-brane module, whose open string excitations correspond to the massless fields of the SM. These SM branes are wrapped around a subset of cycles in the internal space. The second module of D-branes constitute an Hidden Sector (HS) which interacts with the SM only gravitationally or by some vector-like messenger fields. The HS D-branes are wrapped around different internal cycles, and the HS is generically required in order to satisfy all tadpole conditions, discussed above. (As we will discuss, there also exist a few models without HS at all.) In addition, the HS often plays an important phenomenological role, it can by responsible for spontaneous supersymmetry breaking in supersymmetric compactifications, where the supersymmetry breakdown is transferred to the SM either by gravitational interactions (gravity mediation) or by gauge interactions (gauge mediation). Moreover, the HS is responsible for moduli stabilization (see section 4) and/or for cosmic inflation in the early universe (see section 5). This scenario can be depicted in the figure 1.

For a realistic orientifold compactifications, we require that the following two conditions on the open string spectrum are satisfied:

- The open string on the SM branes lead fields of the SM with gauge bosons of the group SU(3) × SU(2)$_L$ × U(1)$_Y$ and three chiral families of quarks and leptons. No other massless (chiral) states are allowed in the SM sector. Hence the massless SM sector is rather model independent from the details of the internal geometry. Each SM field possesses in general a tower of massive string excitations (Regge excitations), which are also independent from the internal geometry, and in addition massive Kaluza Klein (KK) and/or winding states. This part of the spectrum does depend
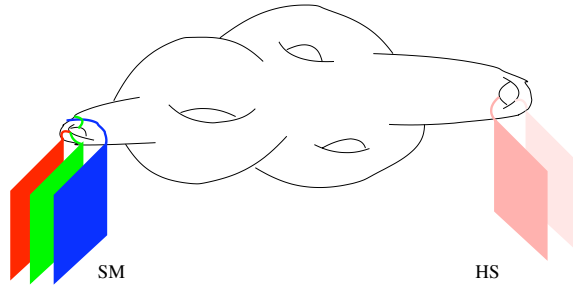
Figure 1. Realization of the SM on a Calabi-Yau space by wrapped D-branes on the left side. The D-branes on the right might be needed for tadpole cancellation and generate a hidden gauge sector.

on the details of the internal geometry. We will discuss in the next section, how SM scattering processes at the LHC can provide rather model independent tests of the SM sector, in particular tests of the Regge spectrum.

• The HS is depends to large extend on the details of the compactification. Its gauge symmetries and massless states are at the moment not further specified. As only constraint on the massless spectrum of the HS we put the condition of *absence of chiral exotics* with SM quantum numbers. This means that there must not be any chiral intersections of the HS branes with the SM branes. However we can allow for vector-like states, which carry both SM and HS quantum numbers. This vector-like states are expected to pair up, such that mass terms for them are generated in the effective potential. Often these vector-like states are phenomenological attractive, since they can act as messenger fields for spontaneous supersymmetry breaking. As a result of these interactions, soft SUSY breaking parameters are generated in the SM sector.

Let us now discuss more specifically the form of the SM brane sector. As emphasized already, one can view this sector as a local D-brane module, which has to be implemented into a global compactification model, i.e. the SM stack of D-branes has to be wrapped around some cycles of the internal space. The massless part of the SM will then arise in a model independent way, as well as its Regge excitations. So this part of the discussion covers a large part of the string landscape, and its possible low energy signatures (see next section) are universal for a large class of point in the landscape. In the following we will describe some local type IIA/IIB D-brane configurations that lead to the SM in a very economic way.
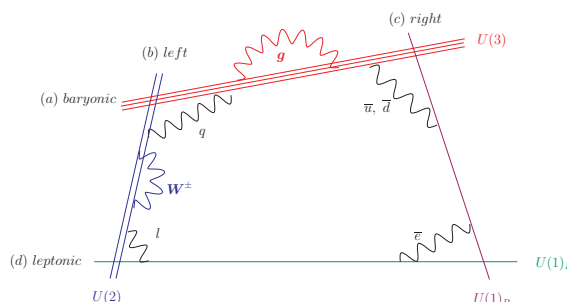
2.3.1 Three stack D-brane models

Here one starts with three stacks of D-branes with initial gauge symmetries:

$$U(3) \times U(2) \times U(1) \times U(1) . \tag{2.8}$$

The (left-handed) SM spectrum is shown in table 2.

matter	$SU(3) \times SU(2) \times U(1)^3$	$U(1)_Y$	$U(1)_{B-L}$
q	$(\mathbf{3}, \mathbf{2})_{(1,1,0)}$	$\frac{1}{3}$	$\frac{1}{3}$
$\bar{u}$	$(\overline{\mathbf{3}}, \mathbf{1})_{(2,0,0)}$	$-\frac{4}{3}$	$-\frac{1}{3}$
$\bar{d}$	$(\overline{\mathbf{3}}, \mathbf{1})_{(-1,0,1)}$	$\frac{2}{3}$	$-\frac{1}{3}$
l	$(\mathbf{1}, \mathbf{2})_{(0,-1,1)}$	-1	-1
$\bar{e}$	$(\mathbf{1}, \mathbf{1})_{(0,2,0)}$	2	1
$\bar{\nu}$	$(\mathbf{1}, \mathbf{1})_{(0,0,-2)}$	0	1

Table 2. Left-handed fermions for the 3 stack model.

Figure 2. A local module of four intersecting stacks of D-branes realizing the SM.

The hypercharge Q_Y is given as the following linear combination of the three U(1)′s:

$$Q_Y = -\frac{2}{3} Q_a + \frac{1}{2} Q_b . \tag{2.9}$$

Here one is forced to realize the left-handed $(\bar{u}, \bar{c}, \bar{t})$-quarks in the antisymmetric representation of U(3), which is the same as the anti-fundamental representation $\overline{3}$. Note that the three stack models with antisymmetric matter are dual to the D3-brane quivers at CY singularities [28–30]. Alternative bottom-up constructions of the SM via D-branes can be found in [31].

2.3.2 Four stack D-brane models

One of the most common ways to realize the SM is by considering four stacks of D-branes. There are several simple ways to embed the SM gauge group into products of unitary and symplectic gauge groups (see [23]). For illustration and also in the next section about LHC signatures, we will use as a prototype model four stacks of D-branes with gauge symmetries:

$$U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d . \tag{2.10}$$

The intersection pattern of the four stacks of D6-branes can be depicted as in figure 2.

The chiral spectrum of the intersecting brane world model should be identical to the chiral spectrum of the SM particles. In type IIA, this fixes uniquely the intersection numbers of the 3-cycles, $(\pi_a, \pi_b, \pi_c, \pi_d)$, the four stacks of D6-branes are wrapped on.

particle	$\mathrm{U}(3)_a \times \mathrm{U}(2)_b \times \mathrm{U}(1)_a \times \mathrm{U}(1)_b \times \mathrm{U}(1)_c \times \mathrm{U}(1)_d$	mult.
q	$(\mathbf{3}, \mathbf{2})_{\mathbf{1}, -\mathbf{1}, \mathbf{0}, \mathbf{0}} + (\mathbf{3}, \mathbf{2})_{\mathbf{1}, \mathbf{1}, \mathbf{0}, \mathbf{0}}$	I_{ab}
$\bar{u}$	$(\bar{\mathbf{3}}, \mathbf{1})_{-\mathbf{1}, \mathbf{0}, -\mathbf{1}, \mathbf{0}} + (\bar{\mathbf{3}}, \mathbf{1})_{-\mathbf{1}, \mathbf{0}, \mathbf{0}, -\mathbf{1}}$	$I_{ac} + I_{ad}$
$\bar{d}$	$(\bar{\mathbf{3}}, \mathbf{1})_{-\mathbf{1}, \mathbf{0}, \mathbf{1}, \mathbf{0}} + (\bar{\mathbf{3}}, \mathbf{1})_{-\mathbf{1}, \mathbf{0}, \mathbf{0}, \mathbf{1}}$	$I_{ac} + I_{ad}$
$\bar{d}'$	$(\bar{\mathbf{3}}_A, \mathbf{1})_{\mathbf{2}, \mathbf{0}, \mathbf{0}, \mathbf{0}}$	$\frac{1}{2} I_{aa}$
l	$(\mathbf{1}, \mathbf{2})_{\mathbf{0}, \mathbf{1}, -\mathbf{1}, \mathbf{0}} + (\mathbf{1}, \mathbf{2})_{\mathbf{0}, \mathbf{1}, \mathbf{0}, -\mathbf{1}}$	$I_{bc} + I_{bd}$
	$+ (\mathbf{1}, \mathbf{2})_{\mathbf{0}, -\mathbf{1}, -\mathbf{1}, \mathbf{0}} + (\mathbf{1}, \mathbf{2})_{\mathbf{0}, -\mathbf{1}, \mathbf{0}, -\mathbf{1}}$	
$\bar{e}$	$(\mathbf{1}, \mathbf{1})_{\mathbf{0}, \mathbf{0}, \mathbf{2}, \mathbf{0}}$	$\frac{1}{2} I_{cc}$
$\bar{e}'$	$(\mathbf{1}, \mathbf{1})_{\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{2}}$	$\frac{1}{2} I_{dd}$
$\bar{e}''$	$(\mathbf{1}, \mathbf{1})_{\mathbf{0}, \mathbf{0}, \mathbf{1}, \mathbf{1}}$	I_{cd}

Table 3. Chiral spectrum for the four stack model with $Q_Y^{(S)}$.

There exist several ways to embed the hypercharge Q_Y into the four U(1) gauge symmetries. The standard electroweak hypercharge $Q_Y^{(S)}$ is given as the following linear combination of three U(1)′s

$$Q_Y^{(S)} = \frac{1}{6} \, Q_a + \frac{1}{2} \, Q_c + \frac{1}{2} \, Q_d \; . \tag{2.11}$$

Therefore, in this case the gauge coupling of the hypercharge is given as

$$\frac{1}{\alpha_Y} = \frac{1}{6} \, \frac{1}{\alpha_a} + \frac{1}{2} \, \frac{1}{\alpha_c} + \frac{1}{2} \, \frac{1}{\alpha_d} \; . \tag{2.12}$$

Now we turn to the particle content of our prototype model. In compact orientifold compactifications each stack of D-branes is accompanied by a orientifold mirror stack of D′-branes. In the next section about the amplitudes, we will not make a difference between the the D-brane and the mirror D′-branes. Hence we will use in the following the indices a, b, c, d collectively for the D-branes as well as for their mirror branes. Then self-intersections among D-branes include intersections between D- and D′-branes. Furthermore, for simplicity, we will suppress from the spectrum those open string states which one also gets from intersections between D-branes and orientifold planes. With these restrictions the left-handed fermion spectrum for our prototype model is presented in table 3.

To derive three generations of quark and leptons, the intersection number in table 4 must satisfy certain phenomenological restrictions: We must have $I_{ab} = 3$. From the left-handed anti u-quarks, we get that $I_{ac} = 3$, and likewise for the two types of left-handed anti d-quarks, we infer that $I_{ac} + I_{ad} + \frac{1}{2} I_{aa} = 3$. In the lepton sector we require that $I_{bc} + I_{bd} = 3$ and $\frac{1}{2}(I_{cc} + I_{dd}) + I_{cd} = 3$.

2.4 Intersecting D6-brane statistics

In this section we first want to count all different, consistent D-brane embeddings into a given closed string geometrical background space. The aim is to find out how many of

them lead to spectrum of the supersymmetric SM. To be specific, we now restrict ourselves on orbifold compactifications, i.e. $\mathcal{M}_6$ is a toroidal Z_N resp. $Z_N \times Z_M$ orientifold. First, we consider the case $\mathcal{M}_6 = T^6/Z_2 \times Z_2 = \prod_{I=1}^3 T_I^2/Z_2 \times Z_2$. The D6-branes are wrapping special Langrangian 3-cycles, which are products of 1-cycles in each of the three subtori T_I^2. Hence they are characterized by three pairs of integer-valued wrapping numbers X^I, Y^I ($I = 0, \ldots, 3$). The supersymmetry conditions, being equivalent to the vanishing of the D-term scalar potential $\mathcal{V}$ have the form:

$$\sum_{I=0}^3 \frac{Y^I}{U_I} = 0\,, \quad \sum_{I=0}^3 X^I U_I > 0\,. \tag{2.13}$$

The U_I are the three complex structure moduli of the three two-tori T_I^2. The Ramond tadpole cancellation conditions for k stacks of N_a D6-branes are given by

$$\sum_{a=1}^k N_a \vec{X}_a = \vec{L}\,, \tag{2.14}$$

where the L^I parametrize the orientifold charge. In addition there are some more constraints from K-theory. Chiral matter in bifundamental representations originate from open strings located at the intersection of two stacks of D6-branes with a multiplicity (generation) number given by the intersection number

$$I_{ab} = \sum_{I=0}^3 (X_a^I Y_b^I - X_b^I Y_a^I)\,. \tag{2.15}$$

2.4.1 $Z_2 \times Z_2$ orientifold

Specifically, we first want to count all different, consistent D-brane embeddings into the given $T^6/Z_2 \times Z_2$ background geometry. i.e. we want to count all possible solutions of the D-brane equations (2.13) and (2.14). These set of equations are diophantic equations in the integer wrapping numbers X^I, Y^I, and they contain as continuous parameters the complex structure moduli U_I. First we want to know, if for any given tadpole charge $\vec{L}$ there is a finite number of solutions of these equations. Actually, based on a saddle point approximation, the total number of D-brane embeddings can be estimated as follows [32]:

$$N_{\text{D-branes}}(L) \simeq e^{2\sqrt{L \log L}}\,. \tag{2.16}$$

For typical orientifold charges like $L = 64$, one obtains as estimate that $N_{\text{D-branes}} \simeq 2 \times 10^9$.

Next, we explicitly count all possible solutions of the D-brane equations (2.13) and (2.14) by running a computer program; this leads to a total of $1.66 \cdot 10^8$ supersymmetric D-brane models on the $Z_2 \times Z_2$ orientifold [33–35]. However this computer count was limited by the available CPU time of about 4×10^5 hours, and hence it could be done only for restricted, not too large values of the complex structure parameters U_I. However, in [36] an analytic proof was found that the number of solutions for eqs. (2.13) and (2.14) is indeed finite.

With this large sample of models we can ask the question which fraction of models satisfy several phenomenological constraints that gradually approach the spectrum of the supersymmetric MSSM.

Restriction	Factor
gauge factor U(3)	0.0816
gauge factor U(2)/Sp(2)	0.992
No symmetric representations	0.839
Massless $U(1)_Y$	0.423
Three generations of quarks ($I_{ab}^{\text{quarks}} = 3$)	2.92×10^{-5}
Three generations of leptons ($I_{ab}^{\text{leptons}} = 3$)	1.62×10^{-3}
Total	1.3×10^{-9}

Table 4. Reduction factors of consistent models with certain properties.

This is summarized in table 4.

The total probability of 1.3×10^{-9} is simply obtained multiplying each probability factors in the first six rows, since one can show that there is little correlation between these individual probabilities. We see that statistically only one in a billion models give rise to an MSSM like D-brane vacuum. Multiplying this result with the initial number of models, the chance to find the MSSM is less then one. One can now compare this statistical result with the explicitly constructed intersecting D6-brane models with MSSM like spectra. In fact, in [37] a $Z_2 \times Z_2$ orientifold model with MSSM like spectrum was found that should be contained in the statistical search discussed above. However unfortunately this model is outside the range of complex structure moduli covered by our computer scan. Also note that all $Z_2 \times Z_2$ MSSM like models found so far contain also chiral exotic particles, not present in the MSSM. These chiral exotic particles can be avoided in the Z_6' orientifolds, as will now discuss in the next subsection.

2.4.2 Z_6 and Z_6' orientifolds

To bypass the problem of getting always chiral exotic massless particles, the statistical scan was extended in [38] to the case of the Z_6 orbifold geometry and in [39–41] to the Z_6' orbifold background geometry. For the first class of orbifold backgrounds explicit MSSM like models were alrady constructed in [42]. Compared to the $Z_2 \times Z_2$ orientifold, the Z_6, Z_6' cases are more complex, because it also contains exceptional, twisted (blowing-up) 3-cycles, besides the untwisted bulk 3-cycles. The D6-branes wrapped around the exceptional 3-cycles correspond to fractional branes. A general fractional cycle can be written with parameters as

$$\Pi^{\text{frac}} = \frac{1}{2}\Pi^{\text{bulk}} + \frac{1}{2}\Pi^{\text{ex}}, \tag{2.17}$$

where we introduced the notation bulk and exceptional cycles for the torus and $\mathbb{Z}_N$ cycles, respectively.

Z_6-orientifold: First, it was possible to show that even in the presence of the exceptional cycles the number of the D-brane solutions of the tadpole plus supersymmetry conditions is very large but nevertheless finite. Then, by extended computer scan it was found that there exist 3.4×10^{28} solutions in total, of which 5.7×10^6 contain the gauge group and the chiral matter content of the MSSM. We therefore obtained a probability of 1.7×10^{-22}
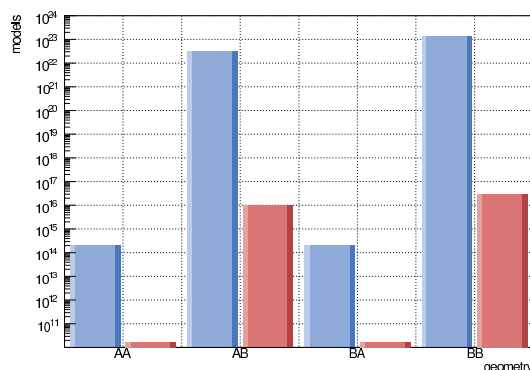
Figure 3. The total number of consistent D6-brane embeddings on the Z'_6 orientifold, depending on different choices of discrete background parameters.
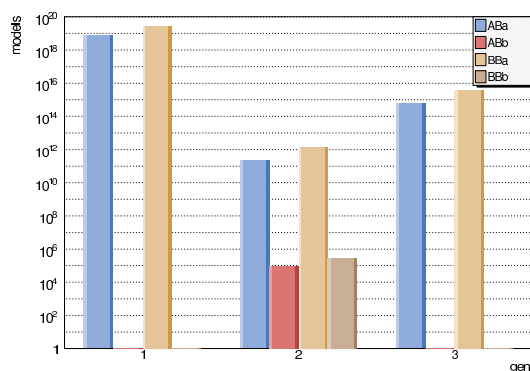
Figure 4. The total number of consistent D6-brane embeddings on the Z'_6 orientifold leading to the three generation SM including chiral exotics

to find MSSM like vacua, a number considerably lower than the value 10^{-9} for the case of the $Z_2 \times Z_2$ orientifolds. However still chiral exotics appear in all solutions.

Z'_6: These are the so far best intersecting D6-brane models seen from the phenomenological point of view. Therefore let us describe the statistical results in more detail.

Table 3 from [40] shows the total number of solutions of the tadpole and supersymmetry conditions which is of order 10^{23}.

The next two figures 4 and 5, again from [40], show numbers of solutions of models on $T^6/\mathbb{Z}'_6$ with SM gauge group and three generations of quarks and leptons. Specifically, in the next figure the total number of three generation models including chiral exotics is shown. In total $\mathcal{O}(10^{19})$ models with three generations have been found.

Finally the total amount of exotic matter in models with three generations is depicted. Asking for zero number of chiral exotics gets an additional suppression factor of $\mathcal{O}(10^{-7})$. Hence, models with only three generations of quarks and leptons and no chiral exotics occur roughly with likelihood of $\mathcal{O}(10^{-11})$, compared to the total number of solutions. Note that
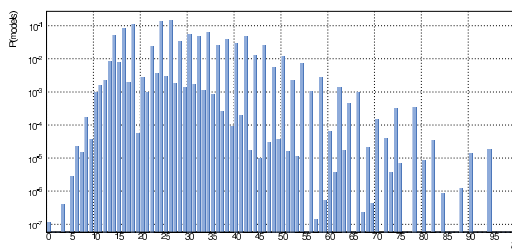
Figure 5. The total number of chiral exotics in Z_6' orientifolds with SM spectra.

most of these models contain a relatively large number of pairs of Higgs doublets, which are not to be confused with chiral exotics, since they are in vector-like representations of the SM gauge group. In addition to the Higgs fields, there are other vector-like exotic states, which will get masses due to some deformations (geometric deformations like blowing up orbifold singularities or also deformations of brane positions) of the background parameters.

3 Possible low energy (LHC) signatures of intersecting D-brane models

3.1 Low string scale in intersecting brane compactifications

There is some good reason to believe that the resolution of the hierarchy problem lies in new physics around the TeV mass scale. The LHC collider at CERN is designed to discover new physics precisely in this energy range, hopefully giving important clues about the nature of dark matter and perhaps at the same time about the solution of the hierarchy problem. In fact, there are at least three, not necessarily mutually exclusive scenarios, offered as solutions of the hierarchy problem:

- Low energy supersymmetry at around 1 TeV.

- New strong dynamics at around 1 TeV (technicolor, little Higgs models, etc).

- Large extra dimensions and a low scale for (quantum) gravity at around 1 TeV.

Here we discuss some universal features of the large extra dimensions scenario [43, 44] relevant for its possible discovery at the LHC. In this scenario, the gravitational and gauge interactions are unified at around 1 TeV, and the observed weakness of gravity at lower energies is due to the existence of large extra dimensions. Gravitons may scatter into the extra space and by this the gravitational coupling constant is decreased to its observed value. Extra dimensions arise naturally in string theory. Hence, one obvious question is how to embed the above scenario into string theory. Then the next important question is what are the possible signatures of large extra dimensions and low gravity in string theory, and how to detect them at the LHC. Large extra dimensions can appear in string theory in case that the intrinsic scale of the string excitations, called the string mass M_{string} is very low, namely at the order of TeV. In this case a whole tower of infinite string excitations

will open up at around 1 TeV, where the new particles essentially follow the well known Regge trajectories of vibrating strings,

$$j = j_0 + \alpha' M^2 \,, \tag{3.1}$$

with the spin j and α' the Regge slope parameter that determines the fundamental string mass scale $M_{\text{string}}^2 = \alpha'^{-1}$.

Let us list what kind of string signatures from a low string scale and from large extra dimensions can be possibly expected at the LHC:

- The discovery of new exotic particles around M_{string}. For example, many string models predict the existence of new, massive Z' gauge bosons from additional U(1) gauge symmetries.

- The discovery of (non-perturbative) quantum gravity effects in the form of mini black holes.

- The discovery of string Regge excitations with masses of order M_{string}.

Before we discuss the above mentioned stringy signatures, we like to describe how large extra dimensions can be realized in Calabi-Yau orientifolds and how the local, SM D-brane system has to be embedded into a large volume Calabi-Yau space.

First we discuss the gravitational and gauge couplings in orientifold compactifications. In the following we consider the type II superstring compactified on a six-dimensional compactification manifold. In addition, we consider a Dp-brane wrapped on a $p-3$-cycle with the remaining four dimensions extended into the uncompactified space-time. We have $d_\parallel = p - 3$ internal directions parallel to the Dp-brane world volume and $d_\perp = 9 - p$ internal directions transverse to the Dp-brane world volume. Let us denote the radii (in the string frame) of the parallel directions by $R_i^\parallel, i = 1, \ldots, d_\parallel$ and the radii of the transverse directions by $R_j^\perp$, $j = 1, \ldots, d_\perp$. While the gauge interactions are localized on the D-brane world volume the gravitational interactions are also spread into the transverse space. This gives qualitatively different quantities for their couplings. In $D = 4$ we obtain for the Planck mass ($\alpha' = M_{\text{string}}^{-2}$)

$$M_{\text{Planck}}^2 = 8 \, e^{-2\phi_{10}} \, M_{\text{string}}^8 \, \frac{V_6}{(2\pi)^6} \,, \tag{3.2}$$

where the internal six-dimensional (string frame) volume V_6 is expressed in terms of the parallel and transversal radii as

$$V_6 = (2\pi)^6 \prod_{i=1}^{d_\parallel} R_i^\parallel \prod_{j=1}^{d_\perp} R_j^\perp \,. \tag{3.3}$$

The dilaton field ϕ_{10} is related to the $D = 10$ type II string coupling constant through $g_{\text{string}} = e^{\phi_{10}}$. The gravitational coupling constant follows from eq. (3.2) through the

relation $\kappa_4^{-2} = \frac{1}{8\pi} M_{\mathrm{Planck}}^2$. On the other hand, in type II superstring theory the gauge theory on the D-brane world-volume has the gauge coupling:

$$g_{Dp}^{-2} = (2\pi)^{-1} \alpha'^{\frac{3-p}{2}} e^{-\phi_{10}} \prod_{i=1}^{d_\parallel} R_i^\parallel \,. \tag{3.4}$$

Here each factor i accounts for an 1-cycle wrapped along the i-th coordinate segment. While the size of the gauge couplings is determined by the size of the parallel dimensions, the strength of gravity is influenced by all directions.

From (3.2) and the gauge coupling (3.4) we may deduce a relation between the Planck mass M_{Planck}, the string mass M_{string} and the sizes R_j of the compactified internal directions. For type II we obtain:

$$g_{Dp}^2 \, M_{\mathrm{Planck}} = 2^{5/2}\pi \, M_{\mathrm{string}}^{7-p} \left(\prod_{j=1}^{d_\perp} R_j^\perp \right)^{\frac{1}{2}} \left(\prod_{i=1}^{d_\parallel} R_i^\parallel \right)^{-1/2} \,. \tag{3.5}$$

Hence, by enlarging some of the transverse compactification radii $R_j^\perp$ the string scale has to become lower in order to achieve the correct Planck mass ($p < 7$). This is to be contrasted with a theory of closed (heterotic) strings only. In that case the relation between the Planck mass and the string scale does not depend on the volume. It is given by the relation $M_{\mathrm{string}} = g_{\mathrm{string}} M_{\mathrm{Planck}}$, which requires a high string scale $M_{\mathrm{string}} \sim 10^{17} \mathrm{GeV}$ for the correct Planck mass.

A priori, there are no compelling reasons why the string mass scale should be much lower than the Planck mass. In the large volume compactifications of [45–47] it was shown that that one can indeed stabilize moduli in such a way that the string scale M_{string} is at intermediate energies of about 10^{11-12} GeV. Then the internal CY volume V is of order $V M_{\mathrm{string}}^6 = \mathcal{O}(10^{16})$. The motivation for this scenario is to obtain a supersymmetry breaking scale around 1 TeV, since one derives the following relation for the gravitino mass:

$$m_{3/2} \sim \frac{M_{\mathrm{string}}^2}{M_{\mathrm{Planck}}} \,. \tag{3.6}$$

However, giving up the requirement of supersymmetry at the TeV scale, one is free to consider CY manifolds with much larger volume. In fact, if it happens for M_{string} to be within the range of LHC energies, not too far beyond 1 TeV, string theory can be tested. In this case the Calabi-Yau volume is as large as $V M_{\mathrm{string}}^6 = \mathcal{O}(10^{32})$. Of course one has to find scalar potentials with minima that lead to such big internal volumes.

Let us now discuss the possible sizes of large extra dimensions subject to the experimental facts. Cavendish type experiments test Newton's law up to a scale of millimeters. This provides an upper bound on the large extra dimensions $R_j^\perp$ to be in the millimeter range. On the other hand, QCD and electroweak scattering experiments give an upper bound on the small extra dimensions $R_i^\parallel$ in the range of the electroweak scale M_{EW}^{-1}.

A first look at the relations (3.2) and (3.3) gives an estimate on the string scale M_{string} and the size of $d_\perp$ extra dimensions $R_j^\perp$. For the $d_\parallel$ small directions to be of the order of

	$d_\perp = 1$	$d_\perp = 2$	$d_\perp = 3$	$d_\perp = 4$	$d_\perp = 5$	$d_\perp = 6$
$R^\perp \ [GeV^{-1}]$	$1.6 \cdot 10^{26}$	$4 \cdot 10^{11}$	$5.4 \cdot 10^{6}$	$2 \cdot 10^{4}$	693	74
$R^\perp \ [m]$	$1.6 \cdot 10^{11}$	$4 \cdot 10^{-4}$	$5.4 \cdot 10^{-9}$	$2 \cdot 10^{-11}$	$7 \cdot 10^{-13}$	$7 \cdot 10^{-14}$
$E_R \ [MeV]$	$7.7 \cdot 10^{-24}$	$3 \cdot 10^{-9}$	$2 \cdot 10^{-4}$	0.06	1	16

Table 5. Size of $d_\perp$ large extra dimensions for a string scale of $M_{\text{string}} = 1 \ TeV$.

the string scale M_{string} and $d_\perp$ extra dimensions of size $R^\perp$ we obtain the values shown in table 5.[1]

So, the case $d_\perp = 1$ is ruled out experimentally.

In fact, it is not completely straightforward to construct SM-like D-brane models on CY spaces with large transverse dimensions. In order to combine D-branes with SM particle content with the scenario of large extra dimensions, one has to consider specific types of Calabi-Yau compactifications. The three or four stacks of intersecting D-branes that give rise to the spectrum of the SM are just local modules that have to be embedded into a global large volume CY-manifold in order to obtain a consistent string compactification. For internal consistency several tadpole and stability conditions have to be satisfied that depend on the details of the compactification, such as background fluxes etc. In this work we will not aim to provide fully consistent orientifold compactifications with all tadpoles cancelled, since it is enough for us to know the properties of the local SM D-brane modules for the computation of the scattering amplitudes among the SM open strings. However it is important to emphasize that in order to allow for large volume compactification, the D-branes eventually cannot be wrapped around untwisted 3- or 4-cycles of a compact torus or of orbifolds, but one has to consider twisted, blowing-up cycles of an orbifold or more general Calabi-Yau spaces with blowing-up cycles. The reason for this is that wrapping the three or four stacks of D-branes around internal cycles of a six-torus or untwisted orbifold cycles, the volumes of these cycles involve the toroidal radii. Therefore these volumes cannot be kept small while making the overall volume of the six-torus very big. Hence, the SM D-branes must be wrapped around small cycles inside a blown up orbifold or a CY manifold. Other cycles have to become large, in order to get a CY space with large volume and a low string scale M_{string}.

Let us here give some short discussion on the volume dependence of the gauge couplings in type IIA orientifolds. The corresponding D6-brane gauge coupling constants are proportional to the volumes of the wrapped 3-cycles, i.e.:

$$g_{D6_a}^{-2} = (2\pi)^{-1} \ \alpha'^{-2} \ \text{Vol}(\Pi_a) \ . \tag{3.7}$$

The volume of the cycle Π_a is given in terms of the associated complex structure moduli U_a of a Calabi-Yau manifold X. To accommodate type IIA orientifolds with low string scale and large overall volume, the corresponding complex structure moduli U_β^s, around which the SM D6-branes are wrapped, must be small compared to the volume of X to

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[1]The above values are computed for $g_{\text{string}} \simeq g^2 = \frac{1}{25}$, i.e. $\alpha = \frac{g^2}{4\pi} = 0.003$. Furthermore, $E_R = \frac{hc}{R^\perp}$ and $1 \ GeV^{-1} \sim 10^{-15}m$.

achieve finite values for the corresponding gauge coupling constants. For this, the Calabi-Yau spaces X must satisfy certain restrictions for large volume compactifications to be possible. In principle the structure of the allowed IIA Calabi-Yau spaces can be inferred from type IIB via mirror symmetry. e.g. one can wrap the D6-branes around certain rigid (twisted) 3-cycles of orbifold compactifications (see e.g. [48]), which can be kept small, whereas the overall volume is made very large.

To perform the computation of the matter field scattering amplitudes, as in type IIB we assume that the 3-cycles, which are are wrapped by the SM D6-branes, are flat and have a kind of toroidal like intersection pattern. Specifically, we assume that the SM sector is wrapped around 3-cycles inside a local $T^2 \times T^2 \times T^2$, and the D6-brane wrappings around the tree 2-tori are described by wrapping numbers (n_a^i, m_a^i) ($i = 1, 2, 3$), where the lengths L_a^i of the wrapped 1-cycles in each T^2 is given by the following equation:

$$L_a^i = \sqrt{(n_a^i)^2 \ (R_i)^2 + (m_a^i)^2 \ (R_{i+1})^2} \ . \tag{3.8}$$

Then the gauge coupling on a D6-brane which is wrapped around a 3-cycle, is:

$$g_{D6_a}^{-2} = (2\pi)^{-1} \ \alpha'^{-3/2} \ e^{-\phi_{10}} \prod_{i=1}^{3} L_a^i \ . \tag{3.9}$$

Here, the 3-cycle Π_a is assumed to be a direct product of three 1-cycles with wrapping numbers (n^i, m^i) w.r.t. a pair of two internal directions.[2] In terms of the corresponding three complex structure moduli U_i of the T^2's this equation becomes

$$g_{D6_a}^{-2} = (2\pi)^{-1} e^{-\phi_4} \prod_{i=1}^{3} \frac{|n_a^i - m_a^i U_i|}{\sqrt{\text{Im}(U_i)}} \ . \tag{3.11}$$

Finally, the intersection angles of the D6-branes with the O6-planes along the three y_i directions can be expressed as

$$\tan(\theta_a^i) = \frac{m_a^i R_{i+1}}{n_a^i R_i} \ , \tag{3.12}$$

and the D6-brane intersection angles are simply given as $\theta_{ab}^i = \theta_b^i - \theta_a^i$. More details about the effective gauge couplings, and also about matter field metrics of these kind of intersecting D-brane models can be found in [127].

3.2 Production of mini black holes at the LHC

One of the most exciting possibilities for the LHC is the discovery of small higher-dimensional black holes that can be formed when two sufficiently energetic particles collide [49–51]. This means that effects of higher dimensional quantum gravity can get strong if the string scale

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[2]In type IIB orientifolds, the gauge coupling of a D7-brane, wrapped around the 4-cycle $T^{2,j} \times T^{2,k}$ with wrapping numbers m^j, m^k and magnetic fluxes f^j, f^k is

$$g_{D7_i}^{-2} = (2\pi)^{-1} \ \alpha'^{-2} \ |m^j m^k| \ \text{Re}(T_j - f^j f^k S) \ . \tag{3.10}$$

is low around the TeV scale, and if the volume of the extra dimensions is large. The geometrical cross section for the production of mini black holes is of the order

$$\sigma(E) \sim \frac{1}{M_{b.h.}^2} \left(\frac{E}{M_{b.h.}} \right)^\alpha, \tag{3.13}$$

where $M_{b.h.}$ is the black hole mass, i.e. the effective scale of quantum gravity, and $\alpha \leq 1$ for higher dimensional black holes. Since the production of mini black holes is basically a non-perturbative effect, the black hole mass is suppressed by the string coupling constant compared to the string scale:

$$M_{b.h.} \sim \frac{M_{\text{string}}}{g_{\text{string}}}. \tag{3.14}$$

Therefore, for weak string coupling, the onset for non-perturbative black hole production is higher than for the production of perturbative Regge excitations, the threshold for an increase in the $2 \to 2$ scattering cross section is almost inevitably lower than the threshold for black hole production (see section 3.4).

3.3 Production of (heavy) Z' gauge bosons and mini-charged particles

Another very interesting signal for new stringy physics at the LHC is the production of heavy neutral Z' gauge bosons (see e.g. [52, 53]). These particles are quite generic in any string compactification, and they receive their mass via a Green-Schwarz mixing with axionic scalar fields. e.g. in the four stack D6-brane model with gauge group $U(3) \times U(2) \times U(1) \times U(1)$ three $U(1)$ gauge bosons will get a mass by the Green-Schwarz effect, and only the hyper charge gauge field related to $U(1)_Y$ stays massless.

To understand the basis of the mechanism giving masses to the $U(1)$'s let us consider the following Lagrangian coupling an Abelian gauge field A_μ to an antisymmetric tensor $B_{\mu\nu}$:

$$\mathcal{L} = -\frac{1}{12} H^{\mu\nu\rho} H_{\mu\nu\rho} - \frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} + \frac{c}{4} \epsilon^{\mu\nu\rho\sigma} B_{\mu\nu} F_{\rho\sigma}, \tag{3.15}$$

where

$$H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\rho B_{\mu\nu} + \partial_\nu B_{\rho\mu}, \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{3.16}$$

and g, c are arbitrary constants. This corresponds to the kinetic term for the fields $B_{\mu\nu}$ and A_μ together with the $B \wedge F$ term. We will now proceed to dualize this Lagrangian in two equivalent ways. First we can re-write it in terms of the (arbitrary) field $H_{\mu\nu\rho}$ imposing the constraint $H = dB$ by the standard introduction of a Lagrange multiplier field η in the following way:

$$\mathcal{L}_0 = -\frac{1}{12} H^{\mu\nu\rho} H_{\mu\nu\rho} - \frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} - \frac{c}{6} \epsilon^{\mu\nu\rho\sigma} H_{\mu\nu\rho} A_\sigma - \frac{c}{6} \eta \epsilon^{\mu\nu\rho\sigma} \partial_\mu H_{\nu\rho\sigma}. \tag{3.17}$$

Notice that integrating out η implies $d^*H = 0$ which in turn implies that (locally) $H = dB$ and then we recover (3.15). Alternatively, integrating by parts the last term in (3.17) we are left with a quadratic action for H which we can solve immediately to find

$$H^{\mu\nu\rho} = -c \, \epsilon^{\mu\nu\rho\sigma} \left(A_\sigma + \partial_\sigma \eta \right). \tag{3.18}$$

Inserting this back into (3.17) we find:

$$\mathcal{L}_A = -\frac{1}{4g^2} F^{\mu\nu} F_{\mu\nu} - \frac{c^2}{2} (A_\sigma + \partial_\sigma \eta)^2 \qquad (3.19)$$

which is just a mass term for the gauge field A_μ after "eating" the scalar η to acquire a mass $m^2 = g^2 c^2$. Notice that this is similar to the Stückelberg mechanism where we do not need a scalar field with a vacuum expectation value to give a mass to the gauge boson, nor do we have a massive Higgs-like field at the end.

In intersecting D6-brane models with four stacks of D-branes, there are four RR two-form fields B_i with couplings to the $U(1)_\alpha$ field strengths:

$$\sum_i c_i^\alpha \, B_i \wedge tr(F^\alpha), \quad i = 1,2,3,4; \quad \alpha = a,b,c,d \qquad (3.20)$$

and in addition there are couplings of the Poincaré dual scalars (representing the same degrees of freedom) η_i of the B_i fields:

$$\sum_i d_i^\beta \eta_i tr(F^\beta \wedge F^\beta), \qquad (3.21)$$

where F^β are the field strengths of any of the gauge groups. The combination of both couplings, by tree-level exchange of the RR-fields, cancels the mixed $U(1)_\alpha$ anomalies $A_{\alpha\beta}$ with any other group G_β as:

$$A_{\alpha\beta} + \sum_i c_i^\alpha d_i^\beta = 0 . \qquad (3.22)$$

The coefficient c_i^α and d_i^α may be computed explicitly for each given D-brane configuration. Now, after a duality transformation the $B \wedge F$ couplings turn into explicit mass terms for the Abelian gauge bosons given by the expression:

$$(M^2)_{\alpha\beta} = g_\alpha g_\beta M_{\text{string}}^2 \sum_{i=1}^3 c_i^\alpha c_i^\beta, \qquad \alpha, \beta = a,b,c,d. \qquad (3.23)$$

where the sum runs over the massive RR-fields present in the models and where g_α is the coupling of $U(1)_\alpha$. Here we have normalized to unity the gauge boson kinetic functions. We see that at weak coupling the masses of the Z' gauge bosons are possibly even lower that the string scale, such that they could be produced at the LHC. Another effect of heavy Z' gauge field is the contribution to the SM ρ-parameter. Finally, anomalous $U1)'$ gauge bosons can also contribute the anomalous magnetic moment of the muon.

Another interesting effect is the mixing of massless or very light Z' gauge bosons in the hidden sector with the standard photon (resp. with the $U(1)_Y$ gauge bosons) by their kinetic energies at one loop string perturbation theory (see e.g. [54]). This effect can by described by a mixing term in the effective low energy action of the form

$$\mathcal{L}_{\text{mix}} = \frac{\chi}{g_a g_b} F_{\mu\nu}^{(a)} F^{(b)\mu\nu} . \qquad (3.24)$$

This is nothing else than an off-diagonal 1-loop string threshold effect due to massive string excitations which carry both electric and also $U(1)'$ gauge charges. If in addition,

the hidden sector contains light hidden sector matter particles, which are charged under $U(1)'$, then these particles also acquire a tiny electric charge of the order

$$Q_e^{(a)} = \chi g_b \,. \tag{3.25}$$

Hence in a wide class of models one can experimentally look for signatures of electrically minicharged particles (MCPs) in high precision experiments. This kind of non-accelerator experiments could provide a very powerful test of the hidden sector in string compactifications.

3.4 Four-point string scattering amplitudes — production of heavy string Regge excitations and KK/winding states

The production of string Regge excitations will lead to new contributions to standard model scattering processes, like QCD jets or scattering of quarks into leptons or gauge bosons, which can be measurable at LHC in case the string scale is low [55–57, 57]. Second there are the KK and winding excitations along the small internal dimensions, i.e. KK and winding excitations of the SM fields. Their masses depend on the internal volumes, and they should be also near the string scale M_{string}.

For those amplitudes involving four gauge bosons or two gauge bosons and two matter fermions, the amplitudes do not depend on the geometry of the underlying Calabi-Yau spaces [57]. This model independence still also holds for the four-fermion matter amplitudes, but only w.r.t. their dependence on the four-dimensional kinematical variables s, t, u. On the other hand, the four-fermion amplitudes do depend on the internal Calabi-Yau geometry and topology. Concretely, the four-fermion amplitudes in general depend on the Calabi-Yau intersection numbers, and also on the rational instanton numbers of the Calabi-Yau space. However, to perform the open string CFT computations for the scattering amplitudes of matter fields we shall assume that the SM D-branes are wrapped around flat, toroidal like cycles. Therefore the four-fermion amplitudes are functions of toroidal wrapping numbers. This sounds in contradiction to what we have stated before about the large volume compactifications. Hence, eventually switching from our toroidal-like results to more general Calabi-Yau expressions, some of the factors, which depend on the toroidal geometry, have to be replaced by geometrical or topological Calabi-Yau parameters. However, the kinematical structure of the matter field amplitudes is universal and not affected by the underlying Calabi-Yau geometry. At any rate, as we shall argue later, for the case that the longitudinal brane directions are somewhat greater than the string scale M_{string} the four-fermion couplings depend only on the local structure of the brane intersections, but not on the global CY geometry.

The general structure of a four point amplitude of four open string states is as follows. Let Φ^i, $i = 1, 2, 3, 4$, represent gauge bosons, quarks of leptons of the standard model realized on three or more stacks of intersecting D-branes. The corresponding string vertex operators V_{Φ^i} are constructed from the fields of the underlying superconformal field theory (SCFT) and contain explicit (group-theoretical) Chan-Paton factors. In order to obtain the scattering amplitudes, the vertices are inserted at the boundary of a disk world-sheet,

and the following SCFT correlation function is evaluated:

$$\mathcal{A}(\Phi^1, \Phi^2, \Phi^3, \Phi^4) = \sum_{\pi \in S_4/Z_2} V_{CKG}^{-1} \int\limits_{\mathcal{I}_\pi} \left(\prod_{k=1}^4 dz_k \right) \langle V_{\Phi^1}(z_1)\, V_{\Phi^2}(z_2)\, V_{\Phi^3}(z_3)\, V_{\Phi^4}(z_4) \rangle \,.$$

(3.26)

Here, the sum runs over all six cyclic inequivalent orderings π of the four vertex operators along the boundary of the disk. Each permutation π gives rise to an integration region $\mathcal{I}_\pi = \{z \in R \,|\, z_{\pi(1)} < z_{\pi(2)} < z_{\pi(3)} < z_{\pi(4)}\}$. The group-theoretical factor is determined by the trace of the product of individual Chan-Paton factors, ordered in the same way as the vertex positions. The disk boundary contains four segments which may be associated to as many as four different stacks of D-branes, since each vertex of a field originating from a D-brane intersection connects two stacks. Thus the Chan-Paton factor may actually contain as many a four traces, all in the fundamental representations of gauge groups associated to the respective stacks. However, purely partonic amplitudes for the scattering of quarks and gluons involve no more than three stacks.

In order to cancel the total background ghost charge of -2 on the disk, the vertices in the correlator (3.26) have to be chosen in the appropriate ghost picture and the picture "numbers" must add to -2. Furthermore, in eq. (3.26), the factor V_{CKG} accounts for the volume of the conformal Killing group of the disk after choosing the conformal gauge. It will be canceled by fixing three vertex positions and introducing the respective c-ghost correlator. Because of the PSL$(2, R)$ invariance on the disk, we can fix three positions of the vertex operators. Depending on the ordering $\mathcal{I}_\pi$ of the vertex operator positions we obtain six partial amplitudes. The first set of three partial amplitudes may be obtained by the choice

$$z_1 = 0\,, \quad z_3 = 1\,, \quad z_4 = \infty\,,$$

(3.27)

while for the second set we choose:

$$z_1 = 1\,, \quad z_3 = 0\,, \quad z_4 = \infty\,.$$

(3.28)

The two choices imply the ghost factor $\langle c(z_1)c(z_2)c(z_3) = z_{13}z_{14}z_{34}\rangle$. The remaining vertex position z_2 takes arbitrary values along the boundary of the disk. After performing all Wick contractions in eq. (3.26) the correlators become basic, and generically for each partial amplitude the integral may be reduced to the Euler Beta function:

$$B(s, u) = \int_0^1 x^{s-1}\,(1-x)^{u-1} = \frac{\Gamma(s)\,\Gamma(u)}{\Gamma(s+u)} = \frac{1}{s} + \frac{1}{u} - \frac{\pi^2}{6}\,(s+u) + \mathcal{O}(\alpha'^2)\,.$$

(3.29)

Due to the extended nature of strings, the world-sheet string amplitudes are generically non-trivial functions in α' in addition to the usual dependence on the kinematic invariants and degrees of freedom of the external states. In the effective field theory description this α'-dependence gives rise to a series of infinite many resonance channels due to Regge excitations and/or new contact interactions. Generically, as we already saw, tree-level string amplitudes involving four gluons or amplitudes with two gluons and two fermions are described by the Euler Beta function depending on the kinematic invariants $s = (k_1 +$
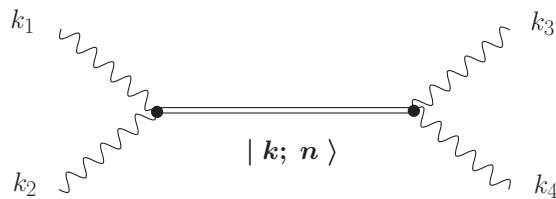
Figure 6. Exchange of an infinite tower of Regge excitations in open string scattering processes.

$k_2)^2$, $t = (k_1 - k_3)^2$, $u = (k_1 - k_4)^2$, with $s + t + u = 0$ and k_i the four external momenta. The whole amplitudes $A(k_1, k_2, k_3, k_4; \alpha')$ may be understood as an infinite sum over s-channel poles with intermediate string states $|k; n\rangle$ exchanged, as it can be seen in the figure 6: After neglecting kinematical factors the string amplitude $A(k_1, k_2, k_3, k_4; \alpha')$ assumes the form

$$A(k_1, k_2, k_3, k_4; \alpha') \sim -\frac{\Gamma(-\alpha' s)\,\Gamma(1 - \alpha' u)}{\Gamma(-\alpha' s - \alpha' u)} = \sum_{n=0}^{\infty} \frac{\gamma(n)}{s - M_n^2} \qquad (3.30)$$

as an infinite sum over s-channel poles at the masses

$$M_n^2 = M_{\text{string}}^2\, n \qquad (3.31)$$

of the string Regge excitations. In eq. (3.30) the residues $\gamma(n)$ are determined by the three-point coupling of the intermediate states $|k; n\rangle$ to the external particles and given by

$$\gamma(n) = \frac{t}{n!}\frac{\Gamma(-u\alpha' + n)}{\Gamma(-u\alpha')} = \frac{t}{n!}\prod_{j=1}^{n}[-u\alpha' - 1 + j] \sim (-\alpha'\, u)^n \quad, \qquad (3.32)$$

with $n + 1$ being the highest possible spin of the state $|k; n\rangle$.

Another way of looking at the expression (3.30) appears, when we express each term in the sum as a power series expansion in α':

$$A(k_1, k_2, k_3, k_4; \alpha') \;\sim\; \underbrace{\frac{t}{s}}_{n=0} \;-\; \underbrace{\frac{\pi^2}{6}\, tu\, \alpha'^2 + \ldots}_{n \neq 0} \, . \qquad (3.33)$$

In this form, the massless state $n = 0$ gives rise to a field-theory contribution ($\alpha' = 0$), while at the order α'^2 all massive states $n \neq 0$ sum up to a finite term. The $n = 0$ term in (3.33) describes the field-theory contribution to the scattering diagram, e.g. the exchange of a massless gluon. On the other hand, the term at the order α'^2 describes a new string contact interaction as a result of summing up all heavy string states. e.g. for a four gluon superstring amplitude the first string contact interaction is given by $\alpha'^2\, g_{Dp}^{-2}\, \text{tr}F^4$, which represent a correction to YM theory, as shown in figure 7.

3.4.1 Four gluon scattering amplitude

Let us start with the open string tree level scattering of four gauge bosons on the disk. The gauge bosons are open strings with ends on same brane, for gluons say the QCD stack

$$\frac{\pi^2}{6}\,\alpha'^2\,\mathrm{tr}F^4$$
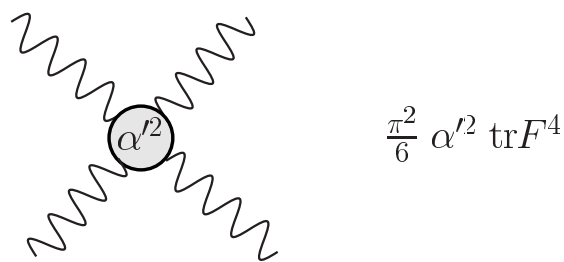
Figure 7. Order $(\alpha')^2$ contact interaction in the scattering of four open string gluons.
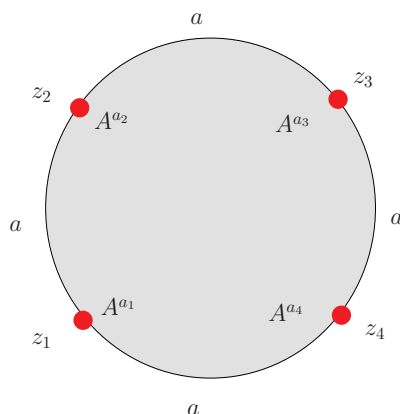
Figure 8. The four gauge boson disk diagram.

a. The gauge boson vertex operator in the (-1)-ghost picture reads

$$V_{A^a}^{(-1)}(z,\xi,k) \;=\; g_A[T^a]_{\alpha_2}^{\alpha_1}\,e^{-\phi(z)}\,\xi^\mu\,\psi_\mu(z)\,e^{ik_\rho X^\rho(z)}\,, \tag{3.34}$$

while in the zero-ghost picture we have:

$$V_{A^a}^{(0)}(z,\xi,k) = \frac{g_A}{(2\alpha')^{1/2}}[T^a]_{\alpha_2}^{\alpha_1}\xi_\mu\,[\;i\partial X^\mu(z) + 2\alpha'\,(k\psi)\,\psi^\mu(z)\;]\,e^{ik_\rho X^\rho(z)}\,. \tag{3.35}$$

where ξ^μ is the polarization vector. The vertex must be inserted on the segment of disk boundary on stack a, with the indices α_1 and α_2 describing the two string ends.

Four-gluon amplitudes have been known for many years. The corresponding string disk diagram is shown in figure 8. The complete amplitude can be generated from the maximally helicity violating MHV amplitudes [59, 60]. Averaging over helicities and colors of the incident partons and summed over helicities and colors of the outgoing particles, we obtain for gluon scattering SU(3)

$$|\mathcal{M}(gg \to gg)|^2 \;=\; \left(\frac{1}{s^2} + \frac{1}{t^2} + \frac{1}{u^2}\right)\left[\frac{9}{4}\big(s^2V_s^2 + t^2V_t^2 + u^2V_u^2\big) - \frac{1}{3}\big(sV_s + tV_t + uV_u\big)^2\right] \tag{3.36}$$

In the D-brane models under consideration, the ordinary SU(3) color gauge symmetry is extended to U(3), so that the open strings terminating on the stack of "color" branes

contain an additional U(1) gauge boson C. Replacing one gluon by the U(1) color singlet gauge boson component C in the QCD stack a, there is also a non-vanishing string amplitude with one photon or one Z-boson ($C = \gamma, Z$), since the photon or the Z-boson always has an admixture of this U(1) gauge group:

$$|\mathcal{M}(gg \to gC)|^2 \; = \frac{5}{6} Q_C^2 \left(\frac{1}{s^2} + \frac{1}{t^2} + \frac{1}{u^2} \right) (\, sV_s + tV_t + uV_u \,)^2 \qquad (3.37)$$

In the zero-slop field theory limit $\alpha' \to 0$ the functions $V_s, V_t, V_u \to 1$, and the four gauge boson amplitudes get contributions only from the exchange of SM fields. In this limit the string amplitudes approach the known results true in the SM Note that the $\mathcal{M}(gg \to gC) \to 0$, as required in the tree level SM. Note that the four gauge boson amplitude is completely model independent, there are no KK-particles being exchanged in the s-channel.

3.4.2 Two gluon, two quark scattering amplitudes

We now consider the following correlation function between two gauge bosons and two matter fermions:

$$\langle V_{A^x}^{(0)}(z_1, \xi_1, k_1) \; V_{A^y}^{(-1)}(z_2, \xi_2, k_2) \; V_{\psi_{\beta_3}^{\alpha_3}}^{(-1/2)}(z_3, u_3, k_3) \; V_{\bar\psi_{\alpha_4}^{\beta_4}}^{(-1/2)}(z_4, \bar u_4, k_4)\rangle \; . \qquad (3.38)$$

The fermion vertex operators are boundary changing operators, being inserted at the intersection of brane stack a and b. Specifically, the chiral fermion vertex operators of the quarks and leptons are:

$$V_{\psi_\beta^\alpha}^{(-1/2)}(z, u, k) = g_\psi [T_\beta^\alpha]_{\alpha_1}^{\beta_1} e^{-\phi(z)/2} \, u^\lambda S_\lambda(z) \, \Xi^{a \cap b}(z) \; e^{ik_\rho X^\rho(z)} \, ,$$
$$V_{\bar\psi_\alpha^\beta}^{(-1/2)}(z, \bar u, k) = g_\psi [T_\alpha^\beta]_{\beta_1}^{\alpha_1} e^{-\phi(z)/2} \, \bar u_{\dot\lambda} S^{\dot\lambda}(z) \, \overline{\Xi}^{a \cap b}(z) \; e^{ik_\rho X^\rho(z)} \, . \qquad (3.39)$$

These vertices connect two segments of disk boundary, associated to stacks a and b, with the indices α_1 and β_1 representing the string ends on the respective stacks. The internal field $\Xi^{a \cap b}$ of conformal dimension $3/8$ is the fermionic boundary changing operator. In the intersecting D-brane models, the intersections are characterized by angles θ_{ba}. Then $\Xi^{a \cap b}$ can be expressed in terms of bosonic and fermionic twist fields σ and s:

$$\Xi^{a \cap b} = \prod_{j=1}^3 \sigma_{\theta_{ba}^j} \; s_{\theta_{ba}^j} \, , \quad \overline{\Xi}^{a \cap b} = \prod_{j=1}^3 \sigma_{-\theta_{ba}^j} \; s_{-\theta_{ba}^j} \, . \qquad (3.40)$$

The spin fields

$$s_{\theta^j} = e^{i(\theta^j - \frac{1}{2})H^j}, \quad s_{-\theta^j} = e^{-i(\theta^j - \frac{1}{2})H^j} \qquad (3.41)$$

have conformal dimension $h_s = \frac{1}{2}(\theta^j - \frac{1}{2})^2$ and twist the internal part of the Ramond ground state spinor. The field σ_θ has conformal dimension $h_\sigma = \frac{1}{2}\theta^j(1 - \theta^j)$ and produces discontinuities in the boundary conditions of the internal complex bosonic Neveu-Schwarz coordinates Z^j.

The fact that fermions originate from the same pair of stacks, say a and b is forced upon us by the conservation of twist charges, in a similar way as their opposite helicities
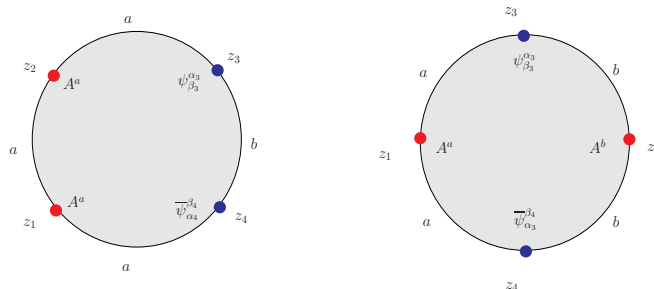
Figure 9. The two gauge boson - two fermion disk diagrams.
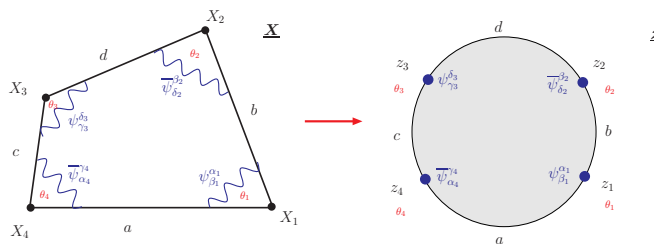
Figure 10. The SM by four stacks of intersecting branes and the corresponding four fermion disk diagram.

are forced by the internal charge conservation. It follows that both gauge bosons must be associated either to one of these stacks, say $(x, y) = (a_1, a_2)$, or one of them is associated to a while the other to b, say $(x, y) = (a, b)$. The corresponding disk diagrams are shown in the figure 9. Using these informations one obtains the following tree level (squared) amplitudes for two gauge boson, two fermion scattering processes [57],

$$|\mathcal{M}(gg \to q\bar{q})|^2 = \frac{t^2 + u^2}{s^2} \left[\frac{1}{6} \frac{1}{ut} (tV_t + uV_u)^2 - \frac{3}{8} V_t V_u \right] \tag{3.42}$$

and

$$|\mathcal{M}(gq \to gq)|^2 = \frac{s^2 + u^2}{t^2} \left[V_s V_u - \frac{4}{9} \frac{1}{su} (sV_s + uV_u)^2 \right] \tag{3.43}$$

Again, in the s-channel there can be only the exchange of heavy Regge states and no KK-states. Hence also these two amplitudes are completely independent from the internal geometry.

3.4.3 Four quark scattering amplitudes

In the most general case, all fermions are at different intersections, as seen in figure 10. Then, without going into further details the corresponding tree level four-fermion scattering

– 27 –

amplitudes take the following form [57]:[3]

$$
\begin{aligned}
|\mathcal{M}(qq \to qq)|^2 &= \frac{2}{9}\frac{1}{t^2}\Big[\big(sF_{tu}^{bb}\big)^2 + \big(sF_{tu}^{cc}\big)^2 + \big(uG_{ts}^{bc}\big)^2 + \big(uG_{ts}^{cb}\big)^2\Big] \\
&+ \frac{2}{9}\frac{1}{u^2}\Big[\big(sF_{ut}^{bb}\big)^2 + \big(sF_{ut}^{cc}\big)^2 + \big(tG_{us}^{bc}\big)^2 + \big(tG_{us}^{cb}\big)^2\Big] \\
&- \frac{4}{27}\frac{s^2}{tu}\big(F_{tu}^{bb}F_{ut}^{bb} + F_{tu}^{cc}F_{ut}^{cc}\big)
\end{aligned}
\tag{3.44}
$$

Here the functions F and G depend on α and now also on the masses of the internal KK (and winding) states. Therefore these amplitudes depend on the internal geometry and are not anymore model independent. Furthermore, due to the quantum numbers of the fermions there are no s-channel poles, but in the t,u-channels there is the exchange of Regge, KK and winding modes, as it can be seen by appropriate expansions of the functions F and G.

3.4.4 Dijet signals for lowest mass strings at the LHC

In this section we will determine the contribution from the exchange of excited, heavy Regge states to dijet processes at the LHC [57]. The first Regge excitations of the gluon (g) and quarks (q) will be denoted by g^*, q^*, respectively. The first excitation of the C will be denoted by C^*. In the following we isolate the contribution to the partonic cross section from the first resonant state. Note that far below the string threshold, at partonic center of mass energies $\sqrt{s} \ll M_s$, the form factor $V(s,t,u) \approx 1 - \frac{\pi^2}{6}su/M_s^4$ and therefore the contributions of Regge excitations are strongly suppressed. The s-channel pole terms of the average square amplitudes contributing to dijet production at the LHC can be obtained from the general formulae given in in the previous subsection. However, for phenomenological purposes, the poles need to be softened to a Breit-Wigner form by obtaining and utilizing the correct *total* widths of the resonances [66]. After this is done, the contributions of the various channels are as follows:

$$
\begin{aligned}
|\mathcal{M}(gg \to gg)|^2 &= \frac{19}{12}\frac{g^4}{M_s^4}\left\{ W_{g^*}^{gg \to gg}\left[\frac{M_s^8}{(s-M_s^2)^2+(\Gamma_{g^*}^{J=0}\ M_s)^2} + \frac{t^4+u^4}{(s-M_s^2)^2+(\Gamma_{g^*}^{J=2}\ M_s)^2}\right] \right. \\
&\left. + W_{C^*}^{gg \to gg}\left[\frac{M_s^8}{(s-M_s^2)^2+(\Gamma_{C^*}^{J=0}\ M_s)^2} + \frac{t^4+u^4}{(s-M_s^2)^2+(\Gamma_{C^*}^{J=2}\ M_s)^2}\right]\right\},
\end{aligned}
\tag{3.45}
$$

$$
\begin{aligned}
|\mathcal{M}(gg \to q\bar{q})|^2 &= \frac{7}{24}\frac{g^4}{M_s^4}\ N_f\left[W_{g^*}^{gg \to q\bar{q}}\frac{ut(u^2+t^2)}{(s-M_s^2)^2+(\Gamma_{g^*}^{J=2}\ M_s)^2} \right. \\
&\left. + W_{C^*}^{gg \to q\bar{q}}\frac{ut(u^2+t^2)}{(s-M_s^2)^2+(\Gamma_{C^*}^{J=2}\ M_s)^2}\right]
\end{aligned}
\tag{3.46}
$$

$$
\begin{aligned}
|\mathcal{M}(q\bar{q} \to gg)|^2 &= \frac{56}{27}\frac{g^4}{M_s^4}\left[W_{g^*}^{q\bar{q} \to gg}\frac{ut(u^2+t^2)}{(s-M_s^2)^2+(\Gamma_{g^*}^{J=2}\ M_s)^2} \right. \\
&\left. + W_{C^*}^{q\bar{q} \to gg}\frac{ut(u^2+t^2)}{(s-M_s^2)^2+(\Gamma_{C^*}^{J=2}\ M_s)^2}\right],
\end{aligned}
\tag{3.47}
$$

---

[3] Four fermion amplitudes in intersecting brane models in the context of proton decay, FCNC currents and Yukawa couolings were also computed in [61–65].
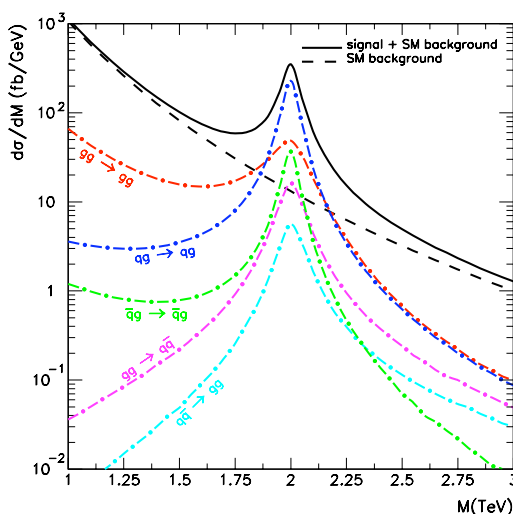
Figure 11. Dijet cross sections.

$$|\mathcal{M}(qg \to qg)|^2 = -\frac{4}{9}\frac{g^4}{M_s^2}\left[\frac{M_s^4 u}{(s-M_s^2)^2+(\Gamma_{q^*}^{J=1/2}\ M_s)^2} + \frac{u^3}{(s-M_s^2)^2+(\Gamma_{q^*}^{J=3/2}\ M_s)^2}\right], \quad (3.48)$$

where g is the QCD coupling constant ($\alpha_{\rm QCD} = \frac{g^2}{4\pi} \approx 0.1$) and $\Gamma_{g^*}^{J=0} = 75\,(M_s/{\rm TeV})$ GeV, $\Gamma_{C^*}^{J=0} = 150\,(M_s/{\rm TeV})$ GeV, $\Gamma_{g^*}^{J=2} = 45\,(M_s/{\rm TeV})$ GeV, $\Gamma_{C^*}^{J=2} = 75\,(M_s/{\rm TeV})$ GeV, $\Gamma_{q^*}^{J=1/2} = \Gamma_{q^*}^{J=3/2} = 37\,(M_s/{\rm TeV})$ GeV are the total decay widths for intermediate states g^*, C^*, and q^* (with angular momentum J) [66]. The associated weights of these intermediate states are given in terms of the probabilities for the various entrance and exit channels

$$W_{g^*}^{gg \to gg} = \frac{(\Gamma_{g^* \to gg})^2}{(\Gamma_{g^* \to gg})^2 + (\Gamma_{C^* \to gg})^2} = 0.09, \quad (3.49)$$

$$W_{C^*}^{gg \to gg} = \frac{(\Gamma_{C^* \to gg})^2}{(\Gamma_{g^* \to gg})^2 + (\Gamma_{C^* \to gg})^2} = 0.91, \quad (3.50)$$

$$W_{g^*}^{gg \to q\bar{q}} = W_{g^*}^{q\bar{q} \to gg} = \frac{\Gamma_{g^* \to gg}\ \Gamma_{g^* \to q\bar{q}}}{\Gamma_{g^* \to gg}\ \Gamma_{g^* \to q\bar{q}} + \Gamma_{C^* \to gg}\ \Gamma_{C^* \to q\bar{q}}} = 0.24, \quad (3.51)$$

$$W_{C^*}^{gg \to q\bar{q}} = W_{C^*}^{q\bar{q} \to gg} = \frac{\Gamma_{C^* \to gg}\ \Gamma_{C^* \to q\bar{q}}}{\Gamma_{g^* \to gg}\ \Gamma_{g^* \to q\bar{q}} + \Gamma_{C^* \to gg}\ \Gamma_{C^* \to q\bar{q}}} = 0.76. \quad (3.52)$$

Superscripts $J = 2$ are understood to be inserted on all the Γ's in eqs. (3.49), (3.50), (3.51), (3.52). Equation (3.45) reflects the fact that weights for $J = 0$ and $J = 2$ are the same [66]. In what follows we set the number of flavors $N_f = 6$.

In figure 11 we show a representative plot of the invariant mass spectrum, for $M_s = 2\,$TeV, detailing the contribution of each subprocess. The QCD background has been calculated at the partonic level from the same processes as designated for the signal, with the addition of $qq \to qq$ and $q\bar{q} \to q\bar{q}$. Our calculation, making use of the CTEQ6D parton distribution functions [67] agrees with that presented in [68]. Finally we estimate (at the
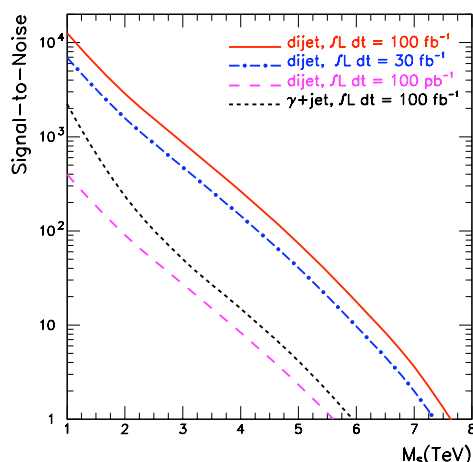
Figure 12. Signal to noise ratio.

parton level) the LHC discovery reach, namely one may calculate a signal-to-noise ratio, with the signal rate estimated in the invariant mass window $[M_s - 2\Gamma, M_s + 2\Gamma]$. The noise is defined as the square root of the number of background events in the same dijet mass interval for the same integrated luminosity. The top two and bottom curves in the next figure show the behavior of the signal-to-noise (S/N) ratio as a function of the string scale for three integrated luminosities ($100\,\text{fb}^{-1}$, $30\,\text{fb}^{-1}$ and $100\,\text{pb}^{-1}$) at the LHC. It is remarkable that within 1-2 years of data collection, *string scales as large as 6.8 TeV are open to discovery at the $\geq 5\sigma$ level.* For $30\,\text{fb}^{-1}$, the presence of a resonant state with mass as large as 5.7 TeV can provide a signal of convincing significance ($S/N \geq 13$). The bottom curve in figure 12, corresponding to data collected in a very early run of $100\,\text{pb}^{-1}$, shows that a resonant mass as large as 4.0 TeV can be observed with 10σ significance! Once more, we stress that these results contain no unknown parameters. They depend only on the D-brane construct for the standard model, and *are independent of compactification details.*

4 Flux compactifications, moduli stabilization and the cosmological constant

In this section we discuss a few aspects about the moduli stabilization process due to background fluxes (see ref. [69, 70] for reviews on flux compactifications) and non-perturbative effects. The number of flux vacua on a given CY background space is very huge [9, 10, 71, 72]: $\mathcal{N}_{\text{vac}} \sim 10^{500}$. This number arises by counting all possible flux combinations that are constrained by satisfying similar tadpole conditions as the D-branes discussed before. Again one can try to make some interesting statistical predictions within the flux landscape, like the question what is the likelihood for obtaining a tiny cosmological constant, or if supersymmetry is broken at high or low energy scales [73–76]. Of course, in order to make more concrete predictions in the string landscape, the flux vacua statistics must be eventually combined with the D-brane statistics, described before. Since the D-branes of SM contribute to the tadpole by a certain amount, the possibilities for introducing back-

grounds are limited in the presence of D-branes. Therefore, if we assume that the SM is present, the number of fluxes is most likely much lower than the $\mathcal{N}_{\text{vac}} \sim 10^{500}$, quoted above. This reduction in flux possibilities should be taken into account when making statistic statements about flux vacua, in particular in connection about the likelihood to obtain a small cosmological constant.

Flux vacua constitute a very interesting region in the string landscape by the following arguments:

- Background fluxes can stabilize moduli. In the description of an effective action, background fluxes generically create a potential for the moduli fields, which leads to a set of discrete vacua. This discretuum of vacua is often called the string landscape, although we have introduced the string landscape os the space of all consistent string solutions. In several cases, the background fluxes do not stabilize all moduli, but additional non-perturbative effects are needed to obtain a discrete landscape of solutions.

- Since many low energy couplings of string compactifications, like gauge couplings or Yukawa couplings are moduli dependent functions, moduli stabilization by fluxes opens at least in principle the possibility to compute these couplings in the string landscape. This is relevant for eventually making contact between the string landscape and the parameters in particle physics, e.g. in the SM.

- The discrete flux vacua (plus possible non-perturbative effects) are a good starting point for string cosmology. Often one needs additional ingredients, like uplift from discrete anti-de Sitter vacua with a negative cosmological constant to de Sitter vacua with a positive cosmological constant. This is also necessary for string inflationary models, where one of the scalar fields plays the role of the inflaton field after moduli stabilization and must have a rather flat, positive effective potential (see section 5).

In this section we will describe some aspects about moduli stabilization in string theory. Massless moduli occur typically in many geometric string compactifications as the parameters, which describe the size and the shape of the internal geometry, as well as the positions of the D-branes in the compact space. In string theoretical language these parameters correspond to marginal conformal fields of conformal dimension $(h, \bar{h}) = 1, 1$. Turning on discrete fluxes the moduli fields become massive and disappear as deformation parameters of the underlying conformal field theory. This effect can be described in the effective, 4-dimensional supergravity description by an effective potential, which fixes the vacuum expectation values of the moduli fields to discrete values and giving them at the same time non-vanishing masses. The associated vacuum energy can be negative (AdS_4 vacua), or positive (dS_4 vacua) or also zero (Minkowski $R^{3,1}$ vacua). In the following we will discuss how moduli stabilization occurs in the effective field theory. We will use the effective superpotential approach (F-terms), neglecting contributions to the scalar potential from D-terms.

After discussing some general and also more mathematical aspects of flux compactifications (see e.g. [69] for a comprehensive review), we will describe type IIB vacua with

fluxes and also possible non-perturbative superpotentials. Then we will focus on AdS_4 vacua in type IIA orientifold compactifications.

4.1 General and mathematical aspects of flux compactifications

The bosonic content of type II supergravity consists of a metric g, a dilaton Φ, an NSNS 3-form flux field H and RR n-form flux fields F_n. In the democratic formalism, where the number of RR-fields is doubled, n runs over $0, 2, 4, 6, 8, 10$ in IIA and over $1, 3, 5, 7, 9$ in type IIB. We write n to denote the dimension of the RR-fields; for example $(-1)^n$ stands for $+1$ in type IIA and -1 in type IIB. After deriving the equations of motion from the action, the redundant RR-fields are to be removed by hand by means of the duality condition:

$$F_n = (-1)^{\frac{(n-1)(n-2)}{2}} e^{\frac{n-5}{2}\Phi} \star_{10} F_{(10-n)} \,, \tag{4.1}$$

given here in the Einstein frame. We will collectively denote the RR-fields, and the corresponding potentials, with polyforms $F = \sum_n F_n$ and $C = \sum_n C_{(n-1)}$, so that: $F = \mathrm{d}_H C$.

In the Einstein frame, the bosonic part of the bulk action reads:

$$S_{\text{bulk}} = \frac{1}{2\kappa_{10}^2} \int \mathrm{d}^{10}x \sqrt{-g} \left[R - \frac{1}{2}(\partial\Phi)^2 - \frac{1}{2}e^{-\Phi}H^2 - \frac{1}{4}\sum_n e^{\frac{5-n}{2}\Phi}F_n^2 \right] \,, \tag{4.2}$$

where for an l-form A we define

$$A^2 = A \cdot A = \frac{1}{l!} A_{M_1\ldots M_l} A_{N_1\ldots N_l} g^{M_1 N_1} \cdots g^{M_l N_l} \,. \tag{4.3}$$

Since (4.1) needs to be imposed by hand this is strictly-speaking only a pseudo-action. Note that the doubling of the RR-fields leads to factors of $1/4$ in their kinetic terms.

The contribution from the calibrated (supersymmetric) brane sources can be written as:

$$S_{\text{source}} = \int \langle C, j \rangle - \sum_n e^{\frac{n}{4}\Phi} \int \langle \Psi_n, j \rangle \,, \tag{4.4}$$

with

$$\Psi_n = e^A \mathrm{d}t \wedge \frac{e^{-\Phi}}{(n-1)!\hat{\epsilon}_1^T \epsilon_1} \hat{\epsilon}_1^{\,T} \gamma_{M_1\ldots M_{n-1}} \hat{\epsilon}_2 \, \mathrm{d}X^{M_1} \wedge \ldots \wedge \mathrm{d}X^{M_{n-1}} \,, \tag{4.5}$$

with $\hat{\epsilon}_{1,2}$ nine-dimensional internal supersymmetry generators, and with the Mukai pairing $\langle \cdot, \cdot \rangle$ given by

$$\langle \phi_1, \phi_2 \rangle = \phi_1 \wedge \alpha(\phi_2)|_{\text{top}} \,. \tag{4.6}$$

The dilaton equation of motion and the Einstein equation read

$$0 = \nabla^2\Phi + \frac{1}{2}e^{-\Phi}H^2 - \frac{1}{8}\sum_n(5-n)e^{\frac{5-n}{2}\Phi}F_n^2 + \frac{\kappa_{10}^2}{2}\sum_n(n-4)e^{\frac{n}{4}\Phi}\star\langle\Psi_n, j\rangle \,, \tag{4.7}$$

$$0 = R_{MN} + g_{MN}\left(\frac{1}{8}e^{-\Phi}H^2 + \frac{1}{32}\sum_n(n-1)e^{\frac{5-n}{2}\Phi}F_n^2\right) \tag{4.8}$$

$$- \frac{1}{2}\partial_M\Phi\partial_N\Phi - \frac{1}{2}e^{-\Phi}H_M \cdot H_N - \frac{1}{4}\sum_n e^{\frac{5-n}{2}\Phi}F_{n\,M} \cdot F_{n\,N}$$

$$- 2\kappa_{10}^2 \sum_n e^{\frac{n}{4}\Phi}\star\langle\left(-\frac{1}{16}n g_{MN} + \frac{1}{2}g_{P(M}dx^P \otimes \iota_{N)}\right)\Psi_n, j\rangle \,,$$

where we defined for an l-form A

$$A_M \cdot A_N = \frac{1}{(l-1)!} A_{MM_2\ldots M_l} A_{NN_2\ldots N_l} g^{M_2 N_2} \cdots g^{M_l N_l} \,. \tag{4.9}$$

The Bianchi identities and the equations of motion for the RR-fields, including the contribution from the 'Chern-Simons' terms of the sources, take the form

$$0 = \mathrm{d}F + H \wedge F + 2\kappa_{10}^2 \, j \,, \tag{4.10}$$

$$0 = \mathrm{d}\left(e^{\frac{5-n}{2}\Phi} \star F_n\right) - e^{\frac{3-n}{2}\Phi} H \wedge \star F_{(n+2)} - 2\kappa_{10}^2 \, \alpha(j) \,. \tag{4.11}$$

Finally, for the equation of motion for H we have:

$$0 = \mathrm{d}(e^{-\Phi} \star H) - \frac{1}{2}\sum_n e^{\frac{5-n}{2}\Phi} \star F_n \wedge F_{(n-2)} + 2\kappa_{10}^2 \sum_n e^{\frac{n}{4}\Phi} \Psi_n \wedge \alpha(j)\bigg|_8 \,. \tag{4.12}$$

For the ten-dimensional metric one uses a general warped ansatz of the form

$$ds^2 = g_{MN}^0 \, dx^M \otimes dx^N = \mathrm{e}^{2\Delta(y)} \left(dx^\mu \otimes dx^\nu \, \hat{g}_{\mu\nu}(x) + dy^m \otimes dy^n \, \hat{g}_{mn}(y)\right) \,. \tag{4.13}$$

The four-dimensional metric $g_{\mu\nu}$ describes either a Minkowski, de Sitter (dS_4), or anti-de Sitter (AdS_4) space. In general other bosonic fields are also allowed to acquire non-trivial profiles and vacuum expectation values in the background, but all fermion fields vanish.

In ten space-time dimensions the type II supersymmetry variations for the two gravitinos ψ_M^A ($A = 1, 2$) and the two dilatinos take the following form (in string frame)

$$\delta\psi_M = \nabla_M \epsilon + \frac{1}{4}\slashed{H}_M \mathcal{P}\epsilon + \frac{1}{16}e^\Phi \sum_n \slashed{F}_n \Gamma_M \mathcal{P}_n \epsilon \,,$$

$$\delta\lambda = \slashed{\partial}\Phi\epsilon + \frac{1}{2}\slashed{H}\mathcal{P}\epsilon + \frac{1}{8}e^\Phi \sum_n (-1)^n(5-n)\slashed{F}_n \mathcal{P}_n \epsilon \,. \tag{4.14}$$

Here the spinors ψ_M, λ and ϵ always combine two spinors but we suppress the label A. The $\mathcal{P}$ and $\mathcal{P}_n$ are 2×2 projection matrices, whose form we do not need explicitly. The vanishing of these variations is required for supersymmetry. The number of Killing spinors ϵ determines the number of supercharges that are preserved. It is now evident that without fluxes and with constant dilaton the solutions are just the covariantly constant spinors of the Calabi-Yau, while fluxes and dilaton profiles turn the conditions into much more complicated looking differential equations.

Now we assume the following $\mathcal{N} = 1$ compactification ansatz for the ten-dimensional supersymmetry generators into four- and six-dimensional spinors:

$$\epsilon_1 = \zeta_+ \otimes \eta_+^{(1)} + \zeta_- \otimes \eta_-^{(1)} \,, \tag{4.15}$$

$$\epsilon_2 = \zeta_+ \otimes \eta_\mp^{(2)} + \zeta_- \otimes \eta_\pm^{(2)} \,, \tag{4.16}$$

for IIA/IIB, where $\zeta_\pm$ are four-dimensional and $\eta_\pm^{(1,2)}$ six-dimensional Weyl spinors. The Majorana conditions for $\epsilon_{1,2}$ imply the four- and six-dimensional reality conditions $(\zeta_+)^* = \zeta_-$ and $(\eta_+^{(1,2)})^* = \eta_-^{(1,2)}$. This reduces the structure of the *generalized* tangent bundle to

SU(3)×SU(3). The structure of the tangent bundle itself on the other hand is a subgroup of SU(3) since there is at least one invariant internal spinor. What subgroup exactly depends on the relation between $\eta^{(1)}$ and $\eta^{(2)}$. Following the terminology of [77, 78] the following classification can be made:

- strict SU(3)-structure: $\eta^{(1)}$ and $\eta^{(2)}$ are parallel everywhere;

- static SU(2)-structure: $\eta^{(1)}$ and $\eta^{(2)}$ are orthogonal everywhere;

- intermediate SU(2)-structure: $\eta^{(1)}$ and $\eta^{(2)}$ at a fixed angle, but neither a zero angle nor a right angle;

- dynamic SU(3)×SU(3)-structure: the angle between $\eta^{(1)}$ and $\eta^{(2)}$ varies, possibly becoming a zero angle or a right angle at a special locus.

Since for static and intermediate SU(2)-structure there are two independent internal spinors the structure of the tangent bundle reduces to SU(2), while for dynamic SU(3)×SU(3)-structure no extra constraints beyond SU(3) are imposed on the topology of the tangent bundle since the two internal spinors $\eta^{(1)}$ and $\eta^{(2)}$ might not be everywhere independent.

SU(3) structure.   Let us consider the SU(3) structure case in more detail (see e.g. [79–83] A globally well defined non-vanishing spinor exists only on manifolds that have *reduced structure*. The structure group of a manifold is the group of transformations required to patch the orthonormal frame bundle. A Riemannian manifold of dimension d has automatically structure group SO(d). All vector, tensor and spinor representations can therefore be decomposed in representations of SO(d). If the manifold has reduced structure group G, then every representation can be further decomposed in representations of G. For $d = 6$, supersymmetry in the absence of fluxes thus leads to the constraint (4.14) implying reduced holonomy for the internal space, from SO(6) $\simeq$ SU(4) to SU(3), so that $\mathcal{X}$ is a Calabi-Yau manifold. The two covariantly constant spinors of type II theories of course lead to $\mathcal{N} = 2$ supersymmetry in four dimensions. With non-vanishing fluxes $\eta_\pm$ can be viewed as covariantly constant with respect to a new connection ∇' different from the Levi-Civita connection. The internal manifold will no longer have SU(3) holonomy (with respect to the Levi-Civita connection). Instead, the requirement of having SU(3) holonomy with respect to the new connection means that the six-dimensional internal manifold has a SU(3) structure group, i.e. the transition functions of the frame bundle take values in an SU(3) $\subset$ SO(6) subgroup. These non-Calabi-Yau space nowadays are characterized by so-called generalized geometry.

The SU(3) group structure allows to decompose vectors, spinors and forms of the internal six-dimensional manifold with respect to their transformation properties under SU(3). This is done by decomposing SO(6) representations in terms of SU(3) representations: $\mathbf{4} \to \mathbf{3} + \mathbf{1}$, $\mathbf{6} \to \mathbf{3} + \bar{\mathbf{3}}$, $\mathbf{15} \to \mathbf{8} + \mathbf{3} + \bar{\mathbf{3}} + \mathbf{1}$, $\mathbf{20} \to \mathbf{6} + \bar{\mathbf{6}} + \mathbf{3} + \bar{\mathbf{3}} + \mathbf{1} + \mathbf{1}$. The spinor representation is in the $\mathbf{4}$ of SO(6) which contains a singlet under SU(3). This means that there exists a globally well defined spinor on the manifold. We furthermore can see that there are also singlets in the decomposition of 2-forms and 3-forms. This means that there is also

a non-vanishing globally well defined real 2-form, and complex 3-form. These are called respectively J and Ω. More precisely, a real non-degenerate two-form J and a complex decomposable three-form Ω completely specify an SU(3)-structure on the six-dimensional manifold $\mathcal{M}$ iff:

$$\Omega \wedge J = 0 \,, \tag{4.17}$$

$$\Omega \wedge \Omega^* = \frac{4i}{3} J^3 \neq 0 \,, \tag{4.18}$$

and the associated metric is positive definite. Up to a choice of orientation, the volume normalization can be taken such that

$$\frac{1}{6} J^3 = -\frac{i}{8} \Omega \wedge \Omega^* = \text{vol}_6 \,. \tag{4.19}$$

When the internal supersymmetry generators of (4.15) are proportional,

$$\eta_+^{(2)} = (b/a) \eta_+^{(1)} \,, \tag{4.20}$$

with $|\eta^{(1)}|^2 = |a|^2, |\eta^{(2)}|^2 = |b|^2$, they define an SU(3)-structure as follows. First let us define a normalized spinor η_+ such that $\eta_+^{(1)} = a\eta_+$ and $\eta_+^{(2)} = b\eta_+$ and moreover we choose the phase of η such that $a = b^*$. Note that in compactifications to AdS$_4$ the supersymmetry imposes $|a|^2 = |b|^2$ such that $b/a = e^{i\theta}$ is just a phase. Now we can construct J and Ω as follows

$$J_{mn} = i\eta_+^\dagger \gamma_{mn} \eta_+ \,, \qquad \Omega_{mnp} = \eta_-^\dagger \gamma_{mnp} \eta_+ \,. \tag{4.21}$$

The intrinsic torsion of $\mathcal{M}$ decomposes into five modules (torsion classes) $\mathcal{W}_1, \ldots, \mathcal{W}_5$. These also appear in the SU(3) decomposition of the exterior derivative of J, Ω. Intuitively, this is because the intrinsic torsion parameterizes the failure of the manifold to be of special holonomy, which can also be thought of as the deviation from closure of J, Ω. In fact, the classification of the different classes of torsion under SU(3) helps in understanding the properties of the underlying geometry. In fact, on a manifold with SU(3) group structure there is always a connection ∇'_m with torsion that has SU(3) holonomy, i.e. $\nabla'_m \eta = 0$. In case the connection is torsionless, the manifold is Calabi-Yau. The torsion tensor can be decomposed in terms of SU(3) representations as follows:

$$\begin{aligned} T^p_{mn} &\in (\mathbf{3} \oplus \bar{\mathbf{3}}) \otimes (\mathbf{1} \oplus \mathbf{3} \oplus \bar{\mathbf{3}}) \\ &= (\mathbf{1} \oplus \mathbf{1}) \oplus (\mathbf{8} \oplus \mathbf{8}) \oplus (\mathbf{6} \oplus \bar{\mathbf{6}}) \oplus 2(\mathbf{3} \oplus \bar{\mathbf{3}}) \\ &\quad\; \mathcal{W}_1 \qquad\quad \mathcal{W}_2 \qquad\quad \mathcal{W}_3 \qquad \mathcal{W}_4, \mathcal{W}_5 \end{aligned} \tag{4.22}$$

$\mathcal{W}_1, \ldots, \mathcal{W}_5$ are the five torsion classes that appear in the covariant derivatives of the spinor, of J and of Ω. $\mathcal{W}_1$ is a complex scalar, $\mathcal{W}_2$ is a complex primitive (1,1) form, $\mathcal{W}_3$ is a real primitive $(2,1) + (1,2)$ form and $\mathcal{W}_4$ and $\mathcal{W}_5$ are real vectors.

The exterior derivative of J and Ω can now be expressed using these torsion classes:

$$\mathrm{d}J = \frac{3}{2} \mathrm{Im}(\mathcal{W}_1 \Omega^*) + \mathcal{W}_4 \wedge J + \mathcal{W}_3 \,, \tag{4.23}$$

$$\mathrm{d}\Omega = \mathcal{W}_1 J \wedge J + \mathcal{W}_2 \wedge J + \mathcal{W}_5^* \wedge \Omega \,, \tag{4.24}$$

where $\mathcal{W}_1$ is a scalar, $\mathcal{W}_2$ is a primitive (1,1)-form, $\mathcal{W}_3$ is a real primitive $(1,2)+(2,1)$-form, $\mathcal{W}_4$ is a real one-form and $\mathcal{W}_5$ a complex (1,0)-form.

A manifold with SU(3) structure is complex if $\mathcal{W}_1 = \mathcal{W}_2 = 0$, i.e. $d\Omega$ is a (3,1)-form. It is symplectic if $\mathcal{W}_1 = \mathcal{W}_3 = \mathcal{W}_4 = 0$, i.e. J is closed. A Kähler manifold is at the same time complex and symplectic, and therefore the only non-zero torsion class can be $\mathcal{W}_5$.[4] Finally for a Calabi-Yau manifold with SU(3) holonomy all five torsion classes are zero.

Geometric fluxes, T-duality and generalized geometry. T-duality is a symmetry of string theory which related string theory on a circle of radius R to a string theory on a dual circle of radius α'/R. Hence it is natural how T-duality acts on flux backgrounds, From the geometrical point of view T-dual backgrounds look rather different, but from the string theory point if view they are equivalent. Also from the low-energy supergravity point of view T-dual backgrounds are seemingly different, in particular two T-dual backgrounds possess in general different SU(3) group structures and torsion classes. Later we will discuss a method to provide a mathematically covariant description of T-duality in terms of generalized SU(3) × SU(3) group structures.

First consider the Ramond fluxes of the type II superstring theories. T-duality (resp. an odd number of T-duality transformations) exchanges the type IIA superstring with the type IIB superstring and vice versa. It follows that the even RR potentials $A^{(n)}$ (n even) of the type IIB superstring are exchanged with the odd RR potentials $A^{(n)}$ (n odd) of the type IIA superstring. So performing a T-duality transformation along the coordinate x, we get the following T-duality rule for the Ramond fluxes:

$$F_{x\alpha_1\cdots\alpha_p}^{(p+1)} \xleftrightarrow{T_x} F_{\alpha_1\cdots\alpha_p}^{(p)}. \tag{4.25}$$

Second, for the universal NS flux field strength $H^{(3)}$ we can use the Buscher rules [84] that were already derived in the world sheet approach. to recall, T-duality in the x direction provides the following new background:

$$G'_{xx} = \frac{1}{G_{xx}}, \qquad G'_{x\mu} = -\frac{B_{x\mu}}{G_{xx}}, \qquad B'_{x\mu} = -\frac{G_{x\mu}}{G_{xx}} \tag{4.26}$$

$$G'_{\mu\nu} = G_{\mu\nu} - \frac{G_{x\mu}G_{x\nu} - B_{x\mu}B_{x\nu}}{G_{xx}} \tag{4.27}$$

$$B'_{\mu\nu} = B_{\mu\nu} - \frac{G_{x\mu}B_{x\nu} - B_{x\mu}G_{x\nu}}{G_{xx}} \tag{4.28}$$

$$e^{\phi'} = \frac{e^{\phi}}{\sqrt{G_{xx}}} \tag{4.29}$$

This basically means that a flux background with non-vanishing $B_{x\mu}$ is T-dualized into a purely geometric back ground with off-diagonal metric $G_{x\mu}$ and vice versa. Switching from B to H, a non-vanishing H_{xyz} gets T-dualized along the x direction in a metric background, which we call $G_{yz} = f_{yz}^x$

$$H_{xyz} \xrightarrow{T_x} f_{yz}^x. \tag{4.30}$$

_____

[4]For a Kähler manifold the Levi-Civita connection has U(3) holonomy.

Since these metric components arise from the NS H-flux after T-duality, one often calls the f_{yz}^x's NS metric, or also geometric fluxes. These constants often appear in so-called twisted tori compactifications [85–87], where they correspond to a certain, underlying algebraic structure.

Let us demonstrate the T-duality transformation rules NS fluxes by an explicit example that of a T^3 with H_3 flux. Note that this background does not satisfy the supersymmetry conditions, since it is a flat background with nontrivial H-flux. This is not a problem, however, as we only use this as an illustrative example and one could e.g. fiber this T^3 over something else to get a good string background. To start, take (x, y, z) as the coordinates on the T^3, each with period 1. Additionally, put N units of H-flux on the torus, such that

$$\int_{T^3} H_3 = N, \tag{4.31}$$

where we have set a pre-factor of $1/(2\pi)^2 \alpha' = 1$ for convenience. To ensure that this quantization condition is satisfied, we can now pick a gauge where $B_{xy} = Nz$, with $N \in \mathbf{Z}$. We have introduced an explicit dependence on the coordinate z. One can view this space not only as a T^3, but also as a T^2 in the (x, y) directions fibered over an S^1 in the z direction, where the Kähler modulus $\rho = (\int B) + iV$ of the T^2 undergoes $\rho \to \rho + N$ as $z \to z + 1$.

Nothing depends on the coordinates x and y, so we can feel free to do a T-duality in either of those directions. T-dualizing on the x direction yields the background

$$ds^2 = (dx - Nz\,dy)^2 + dy^2 + dz^2; \qquad B = 0. \tag{4.32}$$

This is exactly the metric

$$ds^2 = (dx - f_{yz}^x\, z\, dy)^2 + dy^2 + dz^2. \tag{4.33}$$

with $f_{yz}^x = N$, so we see that this background is a twisted torus. In order to make this metric globally well-defined, we need to identify $(x, y, z) \sim (x + Ny, y, z + 1)$. Thus, we see that a T-duality takes $H_{xyz} \xrightarrow{T_x} f_{yz}^x$.

One can easily picture this space as a T^2 in the (x, y) directions fibered over an S^1 in the z direction. As one goes around the S^1 base, the fiber T^2 undergoes a shift in complex structure $\tau \to \tau + f_{yz}^x$. If we want to end up with an equivalent fiber after traversing the S^1, we need to ensure that this is an SL$(2, \mathbf{Z})$ transformation, so we require $f_{yz}^x \in \mathbf{Z}$. This is as expected: one T-duality has switched the complex structure and Kähler moduli.

There is a very useful way of thinking about the number f_{yz}^x. Define the globally invariant one-forms

$$\eta^x = dx - f_{yz}^x\, z\, dy \tag{4.34}$$

$$\eta^y = dy \tag{4.35}$$

$$\eta^z = dz. \tag{4.36}$$

Clearly, $d\eta^y = d\eta^z = 0$, but

$$d\eta^x = f_{yz}^x\, dy \wedge dz = f_{yz}^x \eta^y \wedge \eta^z. \tag{4.37}$$

The f^x_{yz} are just components of the spin connection, by Cartan's structure equations. Additionally, they are the structure constants of a Lie group, which show up as above when the Lie group is viewed as a manifold. This can be easily generalized by considering a manifold with a basis of globally defined one-forms e^a. The generalization of the above construction is that we can write

$$de^a = f^a_{bc} e^b \wedge e^c, \tag{4.38}$$

with the f^a_{bc} are all constant. Note that the requirement that f^a_{bc} be constant is a nontrivial constraint on the manifold. Additionally, the f^a_{bc} must also obey a constraint:

$$d^2 e^a = 2 f^a_{b[c} f^b_{de]} e^d e^e e^c = 0. \tag{4.39}$$

Therefore, the f^a_{bc} obey a Jacobi identity $f^a_{b[c} f^b_{de]} = 0$, as the structure constants of a Lie algebra should. For compact spaces one has to require that $f^a_{ab} = 0$ (no sum); this comes from requiring $d(\alpha e^1 \wedge \ldots \wedge e^{d-1}) = 0$. f^a_{bc} that form a nilpotent algebra automatically satisfy this condition. Such manifolds are called nilmanifolds, or twisted tori, as we will shortly discuss.

Before we come to the next example, let us also mention that T-duality not always lead to a geometrical background [88–90] (for a review on non-geometrical backgrounds see [91]). This can be seen as follows. Starting from the metric (4.33) we still have another T-duality we can do here, since nothing depends on the y direction. The Buscher rules now give

$$ds^2 = \frac{1}{1 + N^2 z^2}(dx^2 + dy^2) + dz^2; \qquad B_{xy} = \frac{Nz}{1 + N^2 z^2}. \tag{4.40}$$

One can check, by examining the Kähler modulus

$$\frac{1}{\rho} = Nz - i, \tag{4.41}$$

that $z \to z + 1$ just takes $1/\rho \to 1/\rho + N$. This is an SL(2, **Z**) transformation on ρ, so once again we see the fiber T^2 shifting its Kähler modulus as we go around the base circle. This is indeed an example of a non-geometric background, since the transformation $1/\rho \to 1/\rho + N$ mixes the metric and the B-field. More precisely, this background is locally geometric, since the metric and B-field are defined at every point, but it is not globally a manifold. Upon going around a cycle, the metric and B-field mix by an SL(2, **Z**) $\subset O(2, 2; **Z**)$ transformation. As with the previous two backgrounds, this one is characterized by an integer N. Writing

$$H_{xyz} \xrightarrow{T_x} f^x_{yz} \xrightarrow{T_y} Q^{xy}_z, \tag{4.42}$$

we will say that this background is characterized by the non-geometric flux Q^{xy}_z. One can show that this object Q^{xy}_z behaves like a one-form, as expected.

Let us study another example, how T-duality acts on a geometrical NS and R flux background, which is actually a solution of the supersymmetry conditions (in addition, one needs also some orientifold charges). We start with a massive IIA solution that is obtained by taking the internal manifold again to be a six-dimensional torus. All torsion classes

vanish in this case. Note, however, that there are non-vanishing H and F_4 fields given by eq. (4.87)

$$H = \frac{2}{5}e^{\Phi}m\left(e^{246} - e^{136} - e^{145} - e^{235}\right) , \tag{4.43}$$

$$F_4 = \frac{3}{5}m\left(e^{1234} + e^{1256} + e^{3456}\right) . \tag{4.44}$$

In addition there is a non-vanishing Romans' mass a parameter m, which can be seen as a 0-form flux (see also section (4.4)):

$$F_0 = \frac{2}{5}m^2 e^{2\Phi} . \tag{4.45}$$

This solution is related, via a single T-duality, to the type IIB superstring on the so-called nilmanifold. Indeed, let us perform a T-duality on the X_6 coordinate of the six-torus example.[5] After rescaling and relabeling the left-invariant forms we find the nilmanifold 5.1 described by the following left-invariant basis of viel-beins:

$$\begin{aligned} \mathrm{d}e^a &= 0, \quad a = 1, \ldots, 5, \\ \mathrm{d}e^6 &= e^{12} + e^{35} . \end{aligned} \tag{4.46}$$

The metric is now given by $g = \mathrm{diag}(1, 1, 1, 1, \beta^2, \beta^2)$, and for the fluxes we have

$$\begin{aligned} H &= -\beta\left(e^{235} + e^{145}\right) , \\ e^{\Phi}F_1 &= \frac{5}{2}\beta^2 e^6 , \\ e^{\Phi}F_3 &= \frac{3}{2}\beta\left(e^{135} - e^{245}\right) , \\ e^{\Phi}F_5 &= \frac{3}{2}\beta^2 e^{12346} . \end{aligned} \tag{4.47}$$

β is related to the mass parameter of the torus example via $\beta = \frac{2}{5}me^{\Phi}$.

Finally, one can perform a second T-duality along the x_5 direction, leading to another geometrical type IIA geometric background, namely the Iwasawa manifold.

At the end of this section we want to discuss the question what is the proper mathematical description of supersymmetric flux backgrounds including T-duality. In particular we have seen that T-duality can lead to backgrounds, which do not allow any more for a description in terms of standard Riemannian geometry, but are rather non-geometrical string backgrounds. One of their key properties is that the transition functions are not anymore diffeomorphisms, but rather T-duality transformations. That means local patches of a T-fold are not glued together by coordinate transformations, but rather by (discrete) T-duality transformations [89, 92]. However, since T-duality is supposed to be a symmetry of string theory, T-fold constitute a class of consistent non-geometrical string backgrounds, like asymmetric orbifolds or general covariant lattice models.

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[5]Note that it does not matter along which direction one performs the T-duality since all six perpendicular directions are equivalent.

The geometric flux backgrounds as well as the non-geometric flux backgrounds can be best described in terms of so-called generalized geometry, which generalize the SU(3) group structures of the geometric backgrounds to the generalized $\text{SU}(3) \times \text{SU}(3)$ group structures [93–95, 99–101]. To understand this recall that in the world sheet approach the closed string is characterized by independent left and right moving coordinates $X_L(\sigma - \tau)$ and $X_R(\sigma + \tau)$. The standard space-time coordinates are given by the sum of the left and right moving coordinates, $X = X_L + X_R$, whereas the dual coordinates are given in terms of their difference: $X^* = X_L - X_R$. Bosonic T-duality transformations on a 6-dimensional background space are given in terms of $\text{SO}(6, 6; \mathbf{Z})$ transformations, which mix the left and right moving degrees of freedom. In fact, the basic T-duality transformation just exchanges $X_L \leftrightarrow X_R$ resp. the position space coordinates with the dual coordinates, i.e. $X \leftrightarrow X^*$. Therefore each closed string excitation not only forms representations of the tangent space of X, called T, but also of the dual tangent space, denoted by T^*. This fact strong suggest to enlarge the space by combining the tangent and the cotangent bundles in a single bundle, $T \oplus T^*$, with generalized structure group $\text{SO}(6, 6)$. Now each (internal) string excitation transforms as a representation of the group $\text{SO}(6, 6)$.

In type II superstrings we have left and right moving internal spinors: $\eta_{\pm}^{(1)}$ for the left moving spinors and $\eta_{\pm}^{(2)}$ for the right moving spinors, where the subscripts $\pm$ denote the two different in-equivalent spinor representations of $\text{SO}(6)$, denoted by $\mathbf{4}_S$ and $\mathbf{f}_C$. Following our strategy to build proper representations of the enlarged frame rotation group $\text{SO}(6, 6)$ it is useful the combine the left and right moving spinors into a single spinor of $\text{SO}(6, 6)$, which is often called pure spinor. Specifically, the supersymmetry generators $\eta^{(1)}$ and $\eta^{(2)}$ from eq. (4.15) are collected into two spinor bilinears, which using the Clifford map, can be associated with two polyforms of definite degree

$$\Psi_+ = \frac{8}{|a||b|} \eta_+^{(1)} \otimes \eta_+^{(2)\dagger}, \qquad \Psi_- = \frac{8}{|a||b|} \eta_+^{(1)} \otimes \eta_-^{(2)\dagger}. \tag{4.48}$$

The subindices plus and minus in $\Phi_{\pm}$ denote the Spin(6,6) chirality: positive corresponds to an even form, and negative to an odd form.

A generalized Calabi-Yau is a manifold on which a closed pure spinor Φ exists with the following property exists:

$$d\Phi = (v_{\llcorner} + \zeta \wedge)\Phi = 0. \tag{4.49}$$

This requirement of having two invariant closed spinors ensures that $\mathcal{N} = 2$ space-time supersymmetry for type II strings and reduces the structure $\text{SO}(6, 6)$ to $\text{SU}(3) \times \text{SU}(3)$. $\Phi_{\pm}$ define therefore an $\text{SU}(3) \times \text{SU}(3)$ group structure on $T \oplus T^*$, similarly as the supersymmetry condition defines an $\text{SU}(3)$ group structure on T for the case of geometrical background spaces. However the existence of an $\text{SU}(3) \times \text{SU}(3)$ group structure is obviously a more general property of supersymmetric string backgrounds, and a $\text{SU}(3)$ group structure does not always exist. T-duality transformations act linearly on the pure spinors $\Phi_{\pm}$, hence $\text{SU}(3) \times \text{SU}(3)$ group structures provide a useful mathematical framework for dealing with T-duality.

In case an SU(3) group structure exist one can relate the pure spinors with the basic geometric objects J and Ω:

$$\Phi_+ = \frac{1}{8}e^{-iJ}\,, \qquad \Phi_- = -\frac{i}{8}\Omega\,. \tag{4.50}$$

Examples of Generalized Calabi-Yau manifolds are symplectic manifolds and complex manifolds with trivial torsion class $\mathcal{W}_5$ (i.e., if $\mathcal{W}_1 = \mathcal{W}_2 = 0$, and $\bar{\mathcal{W}}_5 = \bar{\partial}f$)- then $\Phi = e^{-f}\Omega$ is closed). T-duality basically rotates Φ_+ into Φ_i, and one can show that the mirror symmetry for flux compactifications exchanges symplectic manifolds with complex manifolds, i.e. mirror symmetry acts as:

$$\Omega \longleftrightarrow e^{-iJ}\,. \tag{4.51}$$

Finally, in order to obtain the pure spinors in IIA and IIB one redefines

$$\Psi_1 = \Psi_\mp\,, \qquad \Psi_2 = \Psi_\pm\,, \tag{4.52}$$

with upper/lower sign for IIA/IIB.

Effective supergravity action. The superpotential W and Kähler potential K of the effective $\mathcal{N} = 1$ supergravity action for compactification on spaces with SU(3) × SU(3) or SU(3) group structures have been derived in various ways in [94, 96–98, 102, 103] (based on earlier work of [104, 105]).

The part of the effective four-dimensional action containing the graviton and the scalars reads:

$$S = \int \mathrm{d}^4x \sqrt{-g_4} \left(\frac{M_P^2}{2} R - M_P^2 K_{i\bar{j}} \partial_\mu \phi^i \partial^\mu \bar{\phi}^{\bar{j}} - V(\phi, \bar{\phi}) \right)\,, \tag{4.53}$$

where M_P is the four-dimensional Planck mass. The scalar potential is given in terms of the superpotential via:[6]

$$V(\phi, \bar{\phi}) = M_P^{-2} e^K \left(K^{i\bar{j}} D_i W D_{\bar{j}} W^* - 3|W|^2 \right)\,, \tag{4.54}$$

where the superpotential in the Einstein frame W reads

$$W = \frac{-i}{4\kappa_{10}^2} \int_M \langle \Psi_2, F + i\,\mathrm{d}_H(\mathrm{Re}\mathcal{T}) \rangle\,, \tag{4.55}$$

and $\langle \cdot, \cdot \rangle$ indicates the Mukai pairing, $\mathrm{Re}\mathcal{T} = e^{-\Phi}\mathrm{Im}\Psi_1$, and Ψ_1 and Ψ_2 are the pure spinors describing the geometry. We can rewrite this as

$$W = \frac{-i}{4\kappa_{10}^2} \int_M \langle \Psi_2 e^{\delta B}, F + i\,\mathrm{d}_H(e^{\delta B}\mathrm{Re}\mathcal{T} - i\delta C) \rangle\,,. \tag{4.56}$$

This shows how the fields organize in complex multiplets $\Psi_2 e^{\delta B}$ and $\mathrm{Re}\mathcal{T} - i\delta C$, which will be clearer in concrete examples.

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[6]In [106] the scalar potential was for general type II SU(3)×SU(3) compactifications directly derived from dimensional reduction of the action.

The Kähler potential reads

$$K = -\ln i \int_M \langle \Psi_2, \bar\Psi_2 \rangle - 2 \ln i \int_M \langle t, \bar t \rangle + 3 \ln(8\kappa_{10}^2 M_P^2), \qquad (4.57)$$

where we defined $t = e^{-\Phi}\Psi_1$. Note that $\mathrm{Re}\,t$ should be thought of as a function of $\mathrm{Im}\,t$ so that t can be seen as (non-holomorphically) dependent on $\mathcal{T}$.

Our aim in the next sections will be to find supersymmetric extrema of the scalar potential V. We must therefore impose

$$F_i(\phi_{\min}) = e^{K/2}(\partial_{\phi_i} W + W \partial_{\phi_i} K)|_{\min} = 0 \; \forall i. \qquad (4.58)$$

4.2   Type IIB flux compactifications — the KKLT scenario

We will start without geometrical fluxes; then the tree-level 3-form flux superpotential in type IIB on a Calabi-Yau 3-fold X is of the standard form [104, 105, 107–109]. It gets two kinds of contributions, namely from Ramond and Neveu-Schwarz 3-form fluxes through 3-cycles of the CY space:

$$\begin{aligned}
W_{\mathrm{IIB}} = W_H + W_F &= \int_X \Omega \wedge \left(F_3^{\mathrm{R}} + S H_3^{\mathrm{NS}}\right) \\
&= e_0 + i e_i U_i + i m_0 F_0(U) + m_i F_i(U) \\
&\quad + i S(a_0 + i a_i U_i + i b_0 F_0(U) + b_i F_i(U)).
\end{aligned} \qquad (4.59)$$

Here Ω is the holomorphic 3-form on the CY space, and F_3^{R} (H_3^{NS}) is the Ramond (Neveu-Schwarz) 3-form field strength field. The U-dependent function $F(U) \equiv F_0(U)$ is the holomorphic prepotential and the $F_i(U)$ are its first derivatives. The e_I, m_I comprise the Ramond 3-form fluxes, whereas the a_I, b_I correspond to the Neveu-Schwarz 3-form fluxes ($I = 0, \ldots, h^{2,1}$). The superpotential W depends on the complex-structure moduli fields U_i ($i = 1, \ldots, h^{2,1}$) and on the dilaton S, whereas it is independent of the Kähler moduli T_m ($m = 1, \ldots, h^{1,1}$).

The supersymmetry conditions eq. (4.58) now translate into the conditions on the complex fluxes 3-form fluxes $G_3 = F_3^R + S H_3^{NS}$. First with applying the supersymmetry condition respect to the complex structure moduli U one gets

$$F_U(\phi_{\min}) = e^{K/2}(\partial_U W + W \partial_U K)|_{\min} = 0 \quad \Rightarrow \quad G_{(1,2)} = 0, \qquad (4.60)$$

where $G_{(1,2)}$ is the (1,2)-Hodge component of the complex 3-form fluxes G_3. Similarly supersymmetry with respect to the dilaton S requires

$$F_S(\phi_{\min}) = e^{K/2}(\partial_S W + W \partial_S K)|_{\min} = 0 \quad \Rightarrow \quad G_{(3,0)} = 0, \qquad (4.61)$$

Finally the supersymmetry condition with respect to the Kähler moduli T implies

$$F_T(\phi_{\min}) = e^{K/2}(\partial_T W + W \partial_T K)|_{\min} = 0 \quad \Rightarrow \quad G_{(0,3)} = 0, \qquad (4.62)$$

In summary, applying all three supersymmetry conditions it follows that the 3-form flux must be a self-dual (2,1)-form in the supersymmetric minimum of the potential:

$$G_{(2,1)}|_{\min} \neq 0 \qquad (4.63)$$

In type IIB the fluxes generate a C_4 tadpole given by

$$N_{\text{flux}} = \int H_3 \wedge F_3 = \sum_{I=0}^{h^{2,1}} a_I m_I + b_I e_I \,. \tag{4.64}$$

This flux number is equivalent to the Ramond charge of D3-branes, and has to be cancelled by external sources, namely by the orientifold O3-planes and an appropriate number of D3-branes. Specifically in addition to the above supersymmetry conditions one gets the following tadpole constraints on the 3-form fluxes:

$$N_{\text{flux}} + N_{D_3} = N_{O_3} \,. \tag{4.65}$$

This condition can be reformulated in F-theory in a more geometrical way, where the O3-planes are related to the Euler number χ of an underlying elliptic Calabi-Yau 4-fold X_4. In F-theory the corresponding Gukov-Vafa-Witten superpotential is given in terms of a 4-form flux G_4:

$$W = \int_{X_4} G_4 \wedge \Omega \,. \tag{4.66}$$

Now the flux has to satisfy the following tadpole cancellation condition

$$L \equiv \frac{1}{2} \int_{X_4} G_4 \wedge G_4 = \frac{\chi(X_4)}{24} - N_{D-3} \,. \tag{4.67}$$

This gives an upper bound on the fluxes,

$$L \leq L_* \,, \tag{4.68}$$

with $L_* = \chi(X_4)/24$.

For a given Calabi-Yau 3-fold like an orbifold (or in F-theory for a given 4-fold) it is possible to construct many concrete examples of supersymmetric type IIB flux vacua. e.g. consider a superpotential of the form [108], as it occurs in toroidal or orbifold compactifications:

$$W_{\mathbb{IIB}} = (p + iqSU_1)(l_2 - il_1U_2 + in_1U_3 - n_2U_2U_3) \,. \tag{4.69}$$

p, q, l_1, l_2, n_2, n_2 parametrize the flux quantum numbers that are constrained by the tadpole condition. For fixed flux quantum numbers there is a unique solution of the supersymmetry condition with zero vacuum energy:

$$SU_1 = -\frac{p}{q}\,, \quad U_2 = \sqrt{\frac{l_1 l_2}{n_1 n_2}}\,, \quad U_3 = \sqrt{\frac{l_2 n_1}{l_1 n_2}} \,. \tag{4.70}$$

Moreover the above tadpole conditions are also useful to estimate the number of maximally possible flux vacua on a certain background space. Following [9, 71, 72] this number can be estimated by the following formula:

$$N_{\text{SUSY}} \simeq \frac{L_*^{2h^{2,1}+2}}{(2h^{2,1}+2)!} \,. \tag{4.71}$$

where the Hodge number $h^{2,1}$ counts of complex structure moduli U. Typical numbers for $h^{2,1}$ and L_* lead to a large number for N_{SUSY}:

$$N_{\text{SUSY}} = \mathcal{O}(10^{500}) \,. \tag{4.72}$$

This is indeed a very huge landscape of flux vacua, from which one can possibly argue that there is a good chance to find vacua (after proper uplift to positive vacuum energy) with a tiny positive cosmological constant of order $\Lambda \simeq 10^{-120} M_{\text{Planck}}^4$. In fact, the vast proliferation of string vacua opens the possibility to explain the smallness of the cosmological constant via the anthropic principle [110]. Combing this flux vacua statistics with the intersecting D-brane statistics of section (2.4) (see also next subsection) would possibly also lead to an anthropic explanation of the SM and its parameters. Whether the anthropic principle is really the proper way to understand the landscape is highly debated among theorists (see e.g. [11, 12]), and the outcome of this discussion still has to waited for. Just note however that in case the SM is realized by D3-branes or by D7-branes with F-flux, L is considerably lower that L_*, such that the actual number of flux vacua is much smaller than the number that follows from eq. (4.71).

For geometrical CY spaces with $h^{1,1} > 0$, the flux superpotential (4.59) fixes the complex moduli U and the dilaton S, however not the Kähler moduli T. They are still left as flat direction of the potential. Moreover, the condition eq. (4.62) implies that the superpotential and hence also the scalar potential vanish in the minimum: $W|_{\text{min}} = V|_{\text{min}} = 0$. So the supersymmetric ground state forms a 4-dimensional Minkowski vacuum, i.e. supersymmetry is not compatible with negative vacuum energy. The problem of the Kähler-moduli perturbative independence can be in principle resolved either by introducing geometrical fluxes, i.e. torsion, hence abandoning the CY structure, or in backgrounds which are nongeometrical and do not possess *at all* Kähler moduli (i.e. $h^{1,1} = 0$ in the framework of CY). In addition, the inclusion of non-perturbative effects in the effective superpotential also can fix the Kähler moduli [111]. This is the so-called KKLT scenario, where the total superpotential contains the contributions from the 3-form fluxes (see eq. (4.59)) plus a non-perturbative contribution that depends on the Kähler moduli fields (and also due to threshold effects on the complex structure moduli):

$$W_{KKLT}(T, U, S) = W_{3-\text{form}}(U, S) + W_{\text{n.p.}}(T, U) \tag{4.73}$$

Now, applying the supersymmetry conditions eq. (4.58) to W_{KKLT} allows also for nonvanishing 3-form flux component $G_{(0,3)}$. In addition, now $W|_{\text{min}} \neq 0$, $V|_{\text{min}} < 0$, and hence the flux vacuum is anti-de Sitter like. In the second step of the KKLT scenario, a positive contribution, possibly due to anti-D3-branes or other effects, is added to scalar potential, which leads to a positive vacuum energy, i.e. a 4-dimensional de Sitter vacuum with broken supersymmetry. The typical KKLT potential as a function of the overall Kähler modulus T before and after the uplift is shown in figure 13. It is then possible to analyze the pattern of the soft SUSY breaking mass parameters in the effective supergravity action with D3/D7-branes, which arises in type IIB flux vacua after supersymmetry breaking [46, 112–119].

The non-perturbative part of the KKLT superpotential is provided by Euclidean D3-instantons [120], which are wrapped around 4-cycles (divisors) D inside $\mathcal{M}_6$, and/or gaug-
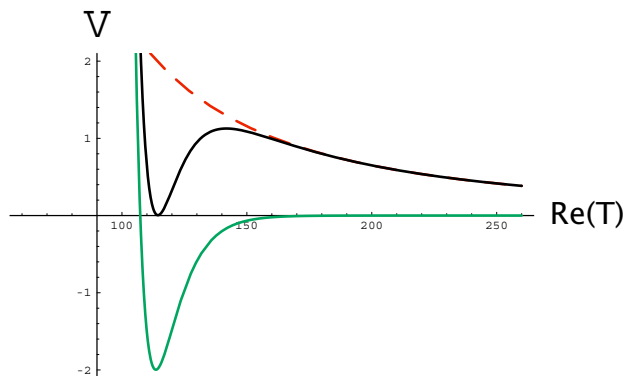
Figure 13. The KKLT potential before the uplift (in red) and after the de Sitter uplift (in black).

ino condensations in hidden gauge group sectors on the world volumes of D7-branes, which are also wrapped around certain divisors D. Both give rise to terms in the superpotential of the form

$$W_{\text{n.p.}}(T,U) \sim g_i(U)\Phi^n e^{-a_i V_i(T)} \,, \tag{4.74}$$

where V_i is the volume of the divisor D_i, depending on the Kähler moduli T, and $g_i(U)$ is a pre-factor, which generically depends on the complex structure moduli U. The fields Φ are matter fields in bifundamental representations that are located at the intersections of space-time filling D7-branes, which are at the same time also intersected by the D3-instantons, resp. the D3-instantons lie on top of the D7-branes.

For gaugino condensation in a hidden gauge group, V_i is the (holomorphic) gauge coupling constant of G_{hidden}, and $W_{\text{n.p.}}$ corresponds to the field theory ADS/TVY superpotential [121, 122]. Here the number of matter fields in $W_{\text{n.p.}}$ is determined by the number of colors and flavors of G_{hidden}. In the simplest case with $G_{\text{hidden}} = \text{SU}(N_C)$ and $N_F = N_C - 1$, $W_{\text{n.p.}}$ is induced by a single D3-instanton that is wrapping the same 4-cycle as the N_c gauge D7-branes. Using these techniques, the moduli stabilization in the KKLT scenario with non-perturbative superpotential was investigated in several orbifold compactifications and their blow-up variants [123–126].

4.3 Combing type IIB flux compactifications and D-brane model building — large volume compactifications

In type IIB orientifolds we assume that the D7-branes are wrapped around 4-cycles inside a CY-orientifold. In the string frame,[7] the volume V_6 of a CY space X is given by

$$V_6 = \frac{1}{3!} \int_X J \wedge J \wedge J = \frac{1}{6} \,\kappa_{ijk}\, t_i t_j t_k \,, \tag{4.75}$$

with t_i ($i = 1, \ldots, h^{1,1}$) the (real) Kähler moduli in the string basis and κ_{ijk} the triple intersection numbers of X. The Kähler form J is expanded w.r.t. a base $\{\hat{D}_i\}$ of the

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[7]In the Einstein frame the Kähler moduli t_k are multiplied by the factor $e^{-\frac{1}{2}\phi_{10}}$. Therefore, in the Einstein frame the CY volume reads $V_6 = \frac{1}{6} e^{-\frac{3}{2}\phi_{10}} \,\kappa_{ijk}\, t_i t_j t_k$.

cohomology $H^{1,1}(X, Z)$ as $J = \sum_{i=1}^{h^{1,1}} t_i \hat{D}_i$. Without loss of generality we restrict to orientifold projections with $h_-^{1,1} = 0$, $h_+^{1,1} = h^{1,1}$. On the other hand, the real parts of the physical Kähler moduli T_i correspond to the volumes of the CY homology four-cycles D_k and are computed from the relation:

$$T_i = \frac{1}{2} \int_{D_i} J \wedge J = \frac{\partial V_6}{\partial t_i} = \frac{1}{2} \kappa_{ijk} t_j t_k \ . \tag{4.76}$$

It follows that the volume V_6 of X becomes a function of degree $3/2$ in the Kähler moduli T_i:

$$V_6 = \frac{1}{3!} \int_X J \wedge J \wedge J = \mathcal{O}(T_i^{3/2}) \ . \tag{4.77}$$

For D7-branes wrapped around the four-cycle D_k, the corresponding gauge coupling constant takes the form[8]

$$g_{D7_k}^{-2} = (2\pi)^{-1} \alpha'^{-2} T_k \ , \tag{4.79}$$

Now we consider CY manifolds which allow for large volume compactification [45–47]. Here one assumes that a set of four-cycles D_α^b ($\alpha = 1, \ldots, h_b^{1,1}$) can be chosen arbitrarily large while keeping the rest of the four-cycles D_β^s ($\beta = 1, \ldots, h^{1,1} - h_b^{1,1}$) small, i.e. $T_\alpha^b \gg T_\beta^s$. Since we want the gauge couplings of the SM gauge groups to have finite, not too small values, we must assume that the SM gauge bosons originate from D7-branes wrapped around the small 4-cycles D_β^s. This splitting of the four-cycles into big and small cycles is only possible, if the CY triple intersection numbers form a specific pattern. In addition, the Euler number of the CY space must be negative, i.e. $h^{2,1} > h^{1,1} > 1$. For a simple class of CY spaces with this property the overall volume V_6 is controlled by one big four-cycle T^b, and the volume has to take the form

$$V_6 \sim (T^b)^{3/2} - h(T_\beta^s) \ , \tag{4.80}$$

where h is a homogeneous function of the small Kähler moduli T_β^s of degree $3/2$. e.g. one may consider the following more specific volume form:

$$V_6 \sim (T^b)^{3/2} - \sum_{\beta=1}^{h^{1,1}-1} (T_\beta^s)^{3/2} \ . \tag{4.81}$$

Looking from the geometrical point of view, these models have a "Swiss cheese" like structures, with holes inside the CY-space given by the small four-cycles.

The simplest example of a Swiss cheese example is the CY manifold $\mathbf{P}_{[1,1,1,6,9]}[18]$ with $h^{1,1} = 2$. In terms of the 2-cycles the volume is given by

$$V_6 = 6 \ (t_1^3 + t_2^3) \ . \tag{4.82}$$

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[8]On the other hands, for (space-time filling) D3-branes the corresponding gauge coupling constant is given by:

$$g_{D3}^{-2} = (2\pi)^{-1} e^{-\phi_{10}} \equiv S \ . \tag{4.78}$$

In the case of magnetic F-fluxes on the D7-brane world-volume the gauge couplings (4.79) receive an additional S-dependent contribution, cf. [127, 128].

According to eq. (4.76) the corresponding 4-cycle volumes become:

$$T^b = \frac{\partial V_6}{\partial t_1} = 18 \, t_1^2 \quad \Longleftrightarrow \quad t_1 = \frac{\sqrt{T^b}}{3\sqrt{2}} \, ,$$

$$T^s = \frac{\partial V_6}{\partial t_2} = 18 \, t_2^2 \quad \Longleftrightarrow \quad t_2 = -\frac{\sqrt{T^s}}{3\sqrt{2}} \, . \tag{4.83}$$

Then the volume can be written in terms of the 4-cycles as

$$V_6 = \frac{1}{9\sqrt{2}} \left[(T^b)^{3/2} - (T^s)^{3/2} \right] . \tag{4.84}$$

So far only discussed the algebraic structure of the CY spaces that allow for large volume compactifications. The next step is then to show that minima with large 4-cycle volumes can be indeed found in the effective potential. This problem was addressed in [46, 47]. They used the standard KKLT 3-form flux superpotential plus the non-perturbative D3-instanton contribution, as given in equation (4.73). In addition, in order to get large four-cycle volumes, perturbative α' corrections have to be included into the tree level Kähler potential, which then reads:

$$K = -2\ln\left(V_6 + \frac{\xi}{2g_s^{3/2}}\right) - \ln\left(S + \bar{S}\right) - \ln\left(-i\int_{CY} \Omega \wedge \bar{\Omega}\right). \tag{4.85}$$

Here the parameter ξ is related to the Euler number χ of the CY-space as follows:

$$\xi = -\frac{\zeta(3)\chi}{2(2\pi)^3} \, . \tag{4.86}$$

Analyzing the structure of the effective potential one can show the minima with large overall volume can indeed occur. Many of the phenomenological properties of large volume compactifications were discussed. Assuming that supersymmetry is spontaneously at an intermediate string scale, $M_{\text{string}} = \mathcal{O}(10^{11}\text{GeV})$ in the hidden sector of the theory, the pattern of the supersymmetry breaking soft terms in the observable sector were discussed in [47, 119]. In [129] a toy model with gauge group $G = \text{U}(N) \times \text{Sp}(2N)$ and with with matter fields on wrapped D7-branes around small cycles and large overall volume was investigated. This model is based on the Swiss cheese example CY manifold $\mathbf{P}_{[1,3,3,3,5]}[15]$ with $h^{1,1} = 3$. Here the conditions for getting a non-perturbative superpotential due to D3-brane instantons and the possibility to have chiral fermions from open strings were discussed in detail. However the matter content of this model was not very realistic, and also not all complex structure moduli could be fixed by the 3-form flux superpotential. Hence, certainly a larger class of large volume CY-spaces has to be investigated in order to find consistent SM realizations by D-branes.

4.4 Type IIA flux compactifications

4.4.1 Type IIA AdS_4 vacua

Now we want to switch from type IIB flux vacua to type IIA flux compactifications to 4-dimensional anti-de Sitter vacua [130–136]. The main motivations to view into the type IIA, AdS_4 flux landscape are the following:

- In type IIA, AdS_4 flux vacua all moduli are generically fixed. As we discuss in the following, one can construct several explicit examples with stabilized moduli in the context of massive type IIA supergravity.

- After a proper uplift to a de Sitter vacuum, type IIA flux compactifications may serve as a good basis for string inflation or obtaining a small cosmological constant. Some aspects of this will be discussed in section (5.3.1).

- Some of the AdS_4 flux vacua can be realized in terms of branes. As we discuss in section 4.5., these branes provide interpolating domain wall solutions between AdS_4 and flat 4-dimensional Minkowski space-time, which may induce transitions between different flux vacua (see section 4.6).

- Type IIA, AdS_4 flux vacua are conjectures to be dual to 3-dimensional Chern-Simons gauge theories, which are conformal field theories in the three dimensions. One known example is type IIA on the background $AdS_4 \times CP^3$ [137]. In would be interesting to determine the 3-dimensional Chern-Simons theories that are dual to massive AdS_4 supergravity, discussed in the following.

In this section we give a short review on the form of the AdS_4 flux vacua in massive IIA supergravity. Up to now all explicit ten-dimensional examples of $\mathcal{N} = 1$ supersymmetric compactifications to AdS$_4$ fall within the class of type IIA SU(3)-structure compactifications and T-duals thereof. The most general form of $\mathcal{N} = 1$ compactifications of IIA supergravity to AdS$_4$ with the ansatz $\eta^{(1)} \propto \eta^{(2)}$ for the internal supersymmetry generators (the strict SU(3)-structure ansatz) was given in [133]. These vacua must have constant warp factor and constant dilaton, Φ. Setting the warp factor to one, the solutions of [133] are given by:

$$H = \frac{2m}{5} e^\Phi \mathrm{Re}\,\Omega \,, \tag{4.87}$$

$$F_2 = \frac{f}{9} J + F_2' \,, \tag{4.88}$$

$$F_4 = f \mathrm{vol}_4 + \frac{3m}{10} J \wedge J \,, \tag{4.89}$$

$$W e^{i\theta} = -\frac{1}{5} e^\Phi m + \frac{i}{3} e^\Phi f \,. \tag{4.90}$$

where H is the NSNS three-form, and F_n denote the RR forms. As before, (J, Ω) is the SU(3)-structure of the internal six-manifold. f, m are constants parameterizing the solution: f is the Freund-Rubin parameter, while m is the mass of Romans' supergravity [138], which can be identified with the type IIA flux F_0. $e^{i\theta}$ is a phase associated with the internal supersymmetry generators: $\eta_+^{(2)} = e^{i\theta} \eta_+^{(1)}$. The constant W is defined by the following relation for the AdS$_4$ Killing spinors, $\zeta_\pm$,

$$\nabla_\mu \zeta_- = \frac{1}{2} W \gamma_\mu \zeta_+ \,, \tag{4.91}$$

so that the radius of AdS$_4$ is given by $|W|^{-1}$. The two-form F_2' is the primitive part of F_2 (i.e. it is in the **8** of SU(3)). It is constrained by the Bianchi identity:

$$\mathrm{d}F_2' = \left(\frac{2}{27}f^2 - \frac{2}{5}m^2\right)e^\Phi \mathrm{Re}\,\Omega - j^6\,, \tag{4.92}$$

where we have added a source, j^6, for D6-branes/O6-planes on the right-hand side.

Finally, the only non-zero torsion classes of the internal manifold are $\mathcal{W}_1^-, \mathcal{W}_2^-$:

$$\mathcal{W}_1^- = -\frac{4i}{9}e^\Phi f\,, \qquad \mathcal{W}_2^- = -ie^\Phi F_2'\,. \tag{4.93}$$

For a given geometry to correspond to a vacuum without orientifold sources, one finds that the following bound on $(\mathcal{W}_1^-, \mathcal{W}_2^-)$ has to be satisfied

$$\frac{16}{5}e^{2\Phi}m^2 = 3|\mathcal{W}_1^-|^2 - |\mathcal{W}_2^-|^2 \geq 0\,. \tag{4.94}$$

The constraint (4.94) can however be relaxed by allowing for an orientifold source, $j^6 \neq 0$. As a particular example, let us consider:

$$j^6 = -\frac{2}{5}e^{-\Phi}\mu\mathrm{Re}\,\Omega\,, \tag{4.95}$$

where μ is an arbitrary, *discrete*, real parameter of dimension (mass)^2, so that $-\mu$ is proportional to the orientifold/D6-brane charge (μ is positive for net orientifold charge and negative for net D6-brane charge). The addition of this source term was also considered in [139, 140]. Eq. (4.95) above guarantees that the calibration conditions, which for D6-branes/O6-planes read

$$j^6 \wedge \mathrm{Re}\,\Omega = 0\,, \qquad j^6 \wedge J = 0\,, \tag{4.96}$$

are satisfied and thus the source wraps supersymmetric cycles. The bound (4.94) changes to

$$e^{2\Phi}m^2 = \mu + \frac{5}{16}\left(3|\mathcal{W}_1^-|^2 - |\mathcal{W}_2^-|^2\right) \geq 0\,. \tag{4.97}$$

Since μ is arbitrary the above equation can always be satisfied, and therefore no longer imposes any constraint on the torsion classes of the manifold.

The corresponding supersymmetric solutions are all of the form $AdS_4 \times X_6$, where X_6 is a certain space which possesses an SU(3) group structure. Recently constructed examples include flat 6-dimensional tori, Nilmanifolds (twisted tori) and several homogenous coset spaces [78, 141–143] In addition to the mass parameter m, several other internal fluxes, like H_3, F_2 and F_4, are also needed in order to preserve supersymmetry, to satisfy all Bianchi identities and to get everywhere regular solutions with finite scalar fields. First, the table 6 shows the coset spaces solutions for massive type IIA supergravity. In the massless limit, the $\mathbb{CP}^3$ coset reduces to the ABJM case, and the supersymmetry is enhanced discontinuously from $\mathcal{N} = 1$ to $\mathcal{N} = 6$.

The other class solutions for massive AdS_4 supergravity require besides the fluxes also orientifold six-planes due to some otherwise uncanceled tadpoles. It is interesting to note

	$\frac{G_2}{SU(3)}$	$\frac{Sp(2)}{S(U(2)\times U(1))}$	$\frac{SU(3)}{U(1)\times U(1)}$	SU(2)×SU(2)	$\frac{SU(3)\times U(1)}{SU(2)}$
Light fields	4	6	8	14	8
Unstabilized	0	0	0	1	0
Decouple KK	no	yes	yes	yes	no
$R < 0$ possible	no	yes	yes	yes	yes

Table 6. Results for the coset spaces.

IIA	IIB	IIA
T^6	nilmanifold 5.1	Iwasawa
D4/D8/NS5	D3/D5/D7/NS5/KK	D2/D6/KK

Table 7. Brane picture.

that these flux geometries also allow for an equivalent brane interpretation of intersecting branes, where the branes act precisely as the sources for the non-vanishing fluxes. In this case, the Romans mass parameter m always corresponds to the presence of m D8-branes. As we will discuss in section (4.5), the flux geometries $AdS_4 \times X_6$ always arise as the near horizon geometry of the corresponding intersecting brane configurations.

A specific massive IIA solution is the first example in table 7, where the internal manifold to be a six-dimensional torus. All torsion classes vanish in this case. Note, however, that there are non-vanishing H and F_4 fields given by (4.87)

$$H = \frac{2}{5}e^{\Phi}m\left(e^{246} - e^{136} - e^{145} - e^{235}\right), \tag{4.98}$$

$$F_4 = \frac{3}{5}m\left(e^{1234} + e^{1256} + e^{3456}\right). \tag{4.99}$$

From (4.97) we find that there is an orientifold source of the type (4.95) with $\mu = e^{2\Phi}m^2$, which corresponds to smeared orientifolds along $(1,3,5)$, $(2,4,5)$, $(2,3,6)$ and $(1,4,6)$. The corresponding orientifold involutions are

$$O6: \quad e^2 \to -e^2, \quad e^4 \to -e^4, \quad e^6 \to -e^6, \tag{4.100}$$

$$O6: \quad e^1 \to -e^1, \quad e^3 \to -e^3, \quad e^6 \to -e^6, \tag{4.101}$$

$$O6: \quad e^1 \to -e^1, \quad e^4 \to -e^4, \quad e^5 \to -e^5, \tag{4.102}$$

$$O6: \quad e^2 \to -e^2, \quad e^3 \to -e^3, \quad e^5 \to -e^5. \tag{4.103}$$

4.4.2 Type IIA effective flux potentials

Now we turn to the 4-dimensional effective action description of type IIA, AdS_4 flux vacua. It was shown for several of the above examples that the spectrum obtained from the effective action matches matches the results from direct dimensional reduction on these spaces [78].

The type IIA effective superpotential receives three kinds of contributions (see e.g. [132, 134–136, 144, 145]):

$$W_{\text{IIA}} = W_H + W_F + W_{\text{geom}}. \tag{4.104}$$

The first term is due to the Neveu-Schwarz 3-form fluxes and depends on the dilaton S and the type IIA complex-structure moduli U_m ($m = 1, \ldots, \tilde{h}^{2,1}$):

$$W_H(S, U) = \int_Y \Omega_c \wedge H_3 = i\tilde{a}_0 S + i\tilde{c}_m U_m \,, \qquad (4.105)$$

where in type IIA the 3-form Ω_c is defined by $\Omega_c = C_3 + i\mathrm{Re}(C\Omega)$. Second, we have the contribution from Ramond 0-, 2-, 4-, 6-form fluxes (the 0-form flux corresponds to the mass parameter $\tilde{m}_0$ in massive IIA supergravity):

$$\begin{aligned}
W_F(T) &= \int_Y \mathrm{e}^{J_c} \wedge F^{\mathrm{R}} \\
&= \tilde{m}_0 \frac{1}{6} \int_Y (J_c \wedge J_c \wedge J_c) + \frac{1}{2} \int_Y \left(F_2^{\mathrm{R}} \wedge J_c \wedge J_c\right) + \int_Y F_4^{\mathrm{R}} \wedge J_c + \int_Y F_6^{\mathrm{R}} \\
&= i\tilde{m}_0 F_0(T) - \tilde{m}_i F_i(T) + i\tilde{e}_i T_i + \tilde{e}_0 \,. \qquad (4.106)
\end{aligned}$$

Here $F(T) := F_0(T)$ is the type IIA prepotential, which depends on the IIA Kähler moduli T_i ($i = 1, \ldots, \tilde{h}^{1,1}$) and $F_i(T) := \partial F_0 / \partial T_i$. We use the notation J_c for the complexified Kähler metric $J_c := B + iJ$. Finally we have the contribution of the geometrical (metric) fluxes, which captures the non-Calabi-Yau property of Y:

$$W_{\mathrm{geom}}(S, T, U) = i \int_Y \Omega_c \wedge \mathrm{d}J = -\tilde{a}_i S T_i - \tilde{d}_{im} T_i U_m \,, \qquad (4.107)$$

where the metric fluxes $\tilde{a}_i$, $\tilde{d}_{im}$ parameterize the non-vanishing of $\mathrm{d}J$.

The type IIA Ramond tadpole follows from the equation of motion of the field C_7. Specifically it is of the form [136]:

$$\tilde{N}_{\mathrm{flux}} = \int \left(C_7 \wedge \mathrm{d}F_2 + C_7 \wedge (\tilde{a}_0 H_3 + \mathrm{d}\bar{F}_2)\right) \,, \qquad (4.108)$$

where $G_2 = \mathrm{d}C_1 + \tilde{a}_0 B_2 + \bar{F}_2$ and $^*F_2 = F_8 = \mathrm{d}C_7$. The metric fluxes $\tilde{a}_i$ contribute to $\mathrm{d}\bar{F}_2$, and one gets for non-vanishing fluxes $\tilde{a}_I$ and $\tilde{m}_I$ that

$$\tilde{N}_{\mathrm{flux}} = \sum_{I=0}^{\tilde{h}^{1,1}} \tilde{a}_I \tilde{m}_I \,. \qquad (4.109)$$

This non-vanishing flux tadpole, which corresponds to a non-vanishing D6-brane charge, must be cancelled by the orientifold O6-planes and an appropriate number of D6-branes:

$$\tilde{N}_{\mathrm{flux}} + N_{\mathrm{D6}} = 2N_{\mathrm{O6}} \,. \qquad (4.110)$$

We now come to the crucial point of generating supersymmetric AdS$_4$ ground states in type IIA with all main moduli stabilized. The superpotential must depend on all chiral fields for the vacuum energy to be negative with unbroken supersymmetry. Following [132, 135, 136] we will concentrate on the case without metric fluxes, i.e. $\tilde{a}_i = \tilde{d}_{im} = 0$. Furthermore, the 6-form fluxes as well as the 2-form fluxes can be shown to be gauge dependent and hence can be set to zero: $\tilde{e}_0 = \tilde{m}_i = 0$ [135, 136]. The fluxes $\tilde{m}_0$ and $\tilde{a}_0$ must be non-zero for W to be kept non-vanishing. Finally, we combine $\tilde{e}_i$ and $\tilde{m}_0$ as

$$\gamma_i = \tilde{m}_0 \tilde{e}_i \,. \qquad (4.111)$$

We will now assume that the internal space Y is simply given by the product of three 2-tori or an orbifold of it. This space has a simple (toroidal) cubic prepotential $F = T_1 T_2 T_3$, and the superpotential has the generic form:

$$W_{\text{IIA}} = W_F + W_H = \tilde{m}_0 \int_Y (J \wedge J \wedge J) + \int_Y F_4^{\text{R}} \wedge J + \int_Y \Omega_c \wedge H_3$$
$$= i\tilde{e}_i T_i + i\tilde{m}_0 T_1 T_2 T_3 + i\tilde{a}_0 S + i\tilde{c}_m U_m \,. \tag{4.112}$$

Using eq. (4.109), the D6-tadpole of corresponding fluxes is simply given by

$$\tilde{N}_{\text{flux}} = \tilde{a}_0 \tilde{m}_0 \,. \tag{4.113}$$

According to eq. (4.110) this number has to be balanced by the D6-branes and the O6-planes.

We may also consider the generalization to the case where the prepotential is given by $F = \frac{1}{6} c_{ijk} T_i T_j T_k$. In CY compactifications, the c_{ijk} would be the classical triple intersection numbers and the corresponding superpotential would read:

$$W_{\text{IIA}} = W_F + W_H = i\tilde{e}_i T_i + i\tilde{m}_0 \frac{1}{6} c_{ijk} T_i T_j T_k + i\tilde{a}_0 S + i\tilde{c}_m U_m \,. \tag{4.114}$$

However, in order to keep the algebra simple we will focus in the following on the toroidal prepotential with $c_{ijk} = 1$.

Coming back to the superpotential (4.112), and Kähler potential $K = -\log(S + \bar{S}) \prod_{i=1}^3 (T_i + \bar{T}_i) \prod_{i=1}^3 (U_i + \bar{U}_i)$, the equations $F_{\phi_i} = 0$ admit the following solution:

$$|\gamma_i| T_i = \sqrt{\frac{5|\gamma_1 \gamma_2 \gamma_3|}{3\tilde{m}_0^2}} \,, \quad S = -\frac{2}{3\tilde{m}_0 \tilde{a}_0} \gamma_i T_i \,, \quad \tilde{c}_m U_m = -\frac{2}{3\tilde{m}_0} \gamma_i T_i \,. \tag{4.115}$$

This solution corresponds to supersymmetric AdS$_4$ vacua. The vacuum energy, i,.e. the AdS_4 cosmological constant is given by the following expression, which entirely depends on that quantized flux quantum numbers;

$$\Lambda_{\text{AdS}} = -3 e^K |W|^2 = -\frac{3^7 \sqrt{\frac{3}{5}}}{100} \frac{|\tilde{a}_0 \tilde{c}_1 \tilde{c}_2 \tilde{c}_3| (|\tilde{m}_0 \tilde{e}_1 \tilde{e}_2 \tilde{e}_3|)^{5/2}}{(\tilde{e}_1 \tilde{e}_2 \tilde{e}_3)^4} M_P^4 \,. \tag{4.116}$$

Let us end the present section by discussing some T-dual/mirror transforms of the of the IIA models. T-duality will in general transform the NS-fluxes into geometrical fluxes. We can for instance investigate within the toroidal models the T-duality transformation in the internal directions x^1 and x^2, acting as $T_1 \to 1/T_1$, $T_{2,3} \to T_{2,3}$. Then the T-dual superpotential of eq. (4.112) becomes

$$W_{\text{IIA}} = \tilde{e}_1 + \tilde{e}_2 T_1 T_2 + \tilde{m}_0 T_2 T_3 + \tilde{e}_3 T_3 T_1 + \tilde{a}_0 S T_1 + \tilde{c}_m T_1 U_m \,. \tag{4.117}$$

The fluxes $\tilde{a}_0$ and $\tilde{c}_m$ become now geometrical. The corresponding AdS$_4$ ground states can be simply obtained by replacing T_1 by $1/T_1$ in eq. (4.115).

Alternatively let us consider the IIB mirror transform of the superpotential (4.112), which is obtained by applying T-duality transformations in the three directions x^1, x^3

and x^5. This exchanges the IIA Kähler moduli by the IIB complex-structure moduli and vice versa. In the presence of the IIA NS-fluxes $\tilde{c}_m$ as in (4.112), the type IIB mirror superpotential will necessarily contain geometrical fluxes:

$$W_{\text{IIB}} = i\tilde{e}_i U_i + i\tilde{m}_0 U_1 U_2 U_3 + i\tilde{a}_0 S + i\tilde{c}_m T_m \,. \tag{4.118}$$

In this case, T-duality takes the system away from the original CY framework.

4.5 AdS_4 domain wall solutions

As we discussed in the previous sections, supersymmetric AdS_4 can appear as ground states of type IIA flux compactifications. Our aim here is precisely to characterize the sources that generate the fluxes necessary for the compactifications under consideration. This complementary, or dual picture, gives another perspective to the emergence of AdS_4. The latter appears as near-horizon geometry of a certain distribution of intersecting/smeared branes and calibrated sources that act as domain walls, connecting AdS_4 to an asymptotically flat region. The brane picture is the first step towards the counting of microscopic states. From the viewpoint of four-dimensional gauged supergravity, one could presumably go further and consider the attractor equations and the macroscopic entropies. This is outside the scope of the present paper.

As we will see explicitly, the appearance of AdS_4 as near-horizon geometry requires that all branes have two common spatial directions in non-compact four-dimensional space-time, i.e. they have the geometry of a domain wall in four dimensions. Moreover, depending on their dimensionality, the branes will fill part of the internal space M_6. Keeping this structure in mind, let us summarize, the relations between the various fluxes and the corresponding source branes [145].

- For a Neveu-Schwarz 3-form flux H_3 through a 3-cycle Σ_3 inside M_6 the sources are NS5-branes wrapped around the dual 3-cycle $\tilde{\Sigma}_3$.

- In the Ramond sector we have fluxes of the Ramond field strengths F_n^{R} through some internal n-dimensional cycles Σ_n. The desired domain-wall configuration in space-time, requires that these fluxes be generated by magnetic brane sources, namely by D$(8-n)$-branes, wrapped around internal cycles $\tilde{\Sigma}_{6-n}$ dual to Σ_n.

- For geometrical fluxes we expect to have Kaluza-Klein monopoles as sources. In fact, performing T-duality to directions orthogonal to the NS5-brane, one obtains a KK-monopole.

The fluxes are quantized and this reflects that the number of branes is not arbitrary.

Any self-consistent system of space-time-filling branes must obey the tadpole cancellation condition, which is a consequence of the (generalized) Gauss law. Alternatively, in supersymmetric configurations, this condition can be thought of as arising from the integrated Bianchi identities. Specifically for the following D4/D8/NS5 example , we will need the D6/O6 tadpole cancellation condition:

$$\frac{1}{2\pi\sqrt{\alpha'}} \int_\Sigma F_0 \, H_3 + N_{\text{D6}} - 2N_{\text{O6}} = 0 \,, \tag{4.119}$$

where $N_{\rm D6}$, $N_{\rm O6}$ is the total number space-time-filling D6-branes, O6-planes wrapping a three-cycle Σ in the internal space.

For these cases it is significant that whenever stabilization is complete, the values of the moduli found by minimizing the scalar potential are recovered by a careful analysis of the space-time background fields near the horizon. In particular the dilaton approaches a finite constant in this limit.

The examples under consideration here are the following:

- Configuration with D4/D8/NS5 branes. This model contains in particular four stacks of intersecting NS5-branes. The background is a IIA ground state of the superpotential (4.112) with *all* terms non-vanishing: $W_{\rm IIA} = i\tilde{e}_i T_i + i\tilde{m}_0 T_1 T_2 T_3 + i\tilde{a}_0 S + i\tilde{c}_m U_m$. This allows for full moduli stabilization (eqs. (4.115)).

- The next model is obtained by performing a T-duality along one direction (say x^1). This is a type IIB model with D3/D5/D7/NS5/KK-branes/monopoles. Its superpotential, generated by F_1, F_3, F_5, H_3 and geometric fluxes, reads $W_{\rm IIB} = i(\tilde{e}_1 U_1 + \tilde{c}_2 U_2 + \tilde{c}_3 U_3 + \tilde{c}_1 T_1 + \tilde{e}_2 T_2 + \tilde{e}_3 T_3 + \tilde{m}_0 U_1 T_2 T_3 + \tilde{a}_0 S)$, and it exhibits an AdS$_4$ vacuum with all moduli stabilized. The latter appears as the near-horizon geometry of the brane/monopole configuration at hand.

Specifically, the first example is given by the following system of intersecting D4/D8/NS5-branes [145]:

	ξ^0	ξ^1	ξ^2	y	x^1	x^2	x^3	x^4	x^5	x^6
D4	$\otimes$	$\otimes$	$\otimes$		$\otimes$	$\otimes$				
D4$'$	$\otimes$	$\otimes$	$\otimes$				$\otimes$	$\otimes$		
D4$''$	$\otimes$	$\otimes$	$\otimes$						$\otimes$	$\otimes$
NS5	$\otimes$	$\otimes$	$\otimes$		$\otimes$		$\otimes$		$\otimes$	
NS5$'$	$\otimes$	$\otimes$	$\otimes$		$\otimes$			$\otimes$		$\otimes$
NS5$''$	$\otimes$	$\otimes$	$\otimes$			$\otimes$		$\otimes$	$\otimes$	
NS5$'''$	$\otimes$	$\otimes$	$\otimes$			$\otimes$	$\otimes$			$\otimes$
D8	$\otimes$	$\otimes$	$\otimes$		$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$

These generate a supersymmetric solution of IIA supergravity in the presence of supersymmetric (calibrated) sources. The tadpole cancellation condition induces in addition O6-planes and/or D6-branes. These brane distributions should thus be referred to as D4/D8/NS5/O6/D6.

In the following we will list a few important facts about this intersecting brane solution:

- The solution is supersymmetric. It can be explicitly given in terms of almost harmonic functions that describe a smearing of the branes over the transverse space.

- The near horizon form of the solution is $AdS_4 \times T^6$. At the other spatial limit the solution just describes $R^{1,3} \times T^6$. Hence this solution is an interpolating domain wall solution between these spaces (see figure 14).
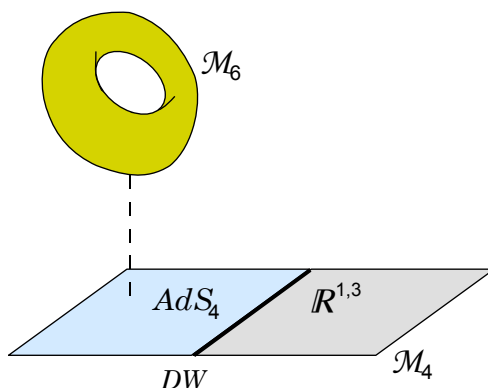
Figure 14. A domain wall separating a region of AdS_4 from a region of $R^{1,3}$. The internal manifold $\mathcal{M}_6$ is fibered over $\mathcal{M}_4$.

- At the horizon we recover that the scalar fields precisely take those fixed values, which were derived for the moduli,in the effective-superpotential description above.

Let us now perform a T-duality along x^1. This results in the following flux superpotential:

$$W_{\text{IIB}} = i\tilde{e}_1 U_1 + i\tilde{c}_2 U_2 + i\tilde{c}_3 U_3 + i\tilde{a}_0 S + i\tilde{c}_1 T_1 + i\tilde{e}_2 T_2 + i\tilde{e}_3 T_3 + i\tilde{m}_0 U_1 T_2 T_3 \,, \qquad (4.120)$$

where the last four terms are the geometrical-flux contributions and guarantee all-moduli stabilization around the type IIB AdS_4 vacuum, as in the type IIA mirror situation.

The corresponding brane configuration is now [145]:

	ξ^0	ξ^1	ξ^2	y	x^1	x^2	x^3	x^4	x^5	x^6
D3	$\otimes$	$\otimes$	$\otimes$			$\otimes$				
D5	$\otimes$	$\otimes$	$\otimes$		$\otimes$		$\otimes$	$\otimes$		
D5$'$	$\otimes$	$\otimes$	$\otimes$		$\otimes$				$\otimes$	$\otimes$
D7	$\otimes$	$\otimes$	$\otimes$			$\otimes$	$\otimes$	$\otimes$	$\otimes$	$\otimes$
NS5	$\otimes$	$\otimes$	$\otimes$		$\otimes$		$\otimes$		$\otimes$	
NS5$'$	$\otimes$	$\otimes$	$\otimes$		$\otimes$			$\otimes$		$\otimes$
KK	$\otimes$	$\otimes$	$\otimes$		$\bullet$	$\otimes$		$\otimes$	$\otimes$	
KK$'$	$\otimes$	$\otimes$	$\otimes$		$\bullet$	$\otimes$	$\otimes$			$\otimes$

Here KK denotes the Kaluza-Klein monopoles that are T-dual to NS5-branes. Note that the near horizon geometry is not anymore a six-dimensional torus, but the T-dual near horizon geometry is that of a *twisted torus*, or a *nilmanifold*, i.e. $AdS_4 \times N_{5,1}$.

4.6 Transitions in the flux landscape

In order to get transitions between vacua with different flux quantum numbers, one needs non-perturbative, gravitational configurations which are coupled to the flux background fields, and which interpolate between different flux vacua. These are given in term of

BPS or nearly BPS domain walls (membranes) (for earlier work see e.g. [146, 147]) in four-dimensional space time that are coupled to the scalar moduli fields. The profile of the domain wall is such that it separates spatial regions with different flux quantum numbers from each other. For the case that the domain wall is interpolating between two supersymmetric vacua, the interpolating solutions is describing a BPS domain wall. Of course, eventually we are interested in the decay of a non-supersymmetric flux vacuum with positive cosmological constant (our vacuum) and broken space-time supersymmetry into another (supersymmetric) flux vacuum, which cam have either positive, zero or also negative cosmological constant (AdS_4) vacuum. The formation of an AdS_4 domain wall is particularly interesting, since AdS_4 are very common in the string landscape. In this case our universe would be decaying into a contracting space, which at first sight seems to be problematic. Nevertheless the corresponding transition amplitude from dS_4 to AdS_4 is expected to be non-vanishing, as it was discussed in [148].

Now let us consider the corresponding the domain wall solution which interpolates between the above AdS_4 flux vacuum and flat Minkowski space-time with vanishing fluxes. As discussed in the previous section it is given in terms of interesting D4,- D8- and NS 5-branes. In addition one also needs orientifold 6-planes (O6) in order to cancel the induces D6-brane charge from the fluxes. The four dimensional part of the metric is such of an interpolating domain wall, where the intersecting branes are smeared in the direction transversal to the domain wall. Specifically, this 4-dimensional part of the metric can be written as

$$ds^2 = a(r)^2(-dt^2 + dx^2 + dy^2) + dr^2 \,. \tag{4.121}$$

For $r \to 0$ this metric approaches the metric of AdS_4, and the scalar fields are fixed to the values determined by the non-vanishing fluxes, as given in eq. (4.115). For $r \to \infty$, the function $a(r)$ becomes a constant, and the eq. (4.121) become the metric of flat Minkowski space.

The tension σ of the domain wall can be computed by introducing a central function $Z(r)$ which is defines as

$$Z(r) = \frac{a'(r)}{a(r)} \,. \tag{4.122}$$

By comparison with the exact metric of [145] one obtains

$$Z(r)|_{r=0} = e^{K/2}|W| \,, \quad \Lambda_{\text{AdS}} = -3|Z(r)|_{r=0}^2 \,. \tag{4.123}$$

The (membrane) tension σ of the domain wall is then given by the following expression:

$$\sigma \simeq (|Z|_{r=\infty} - |Z|_{r=0}) \,. \tag{4.124}$$

Now let us determine the decay amplitude of the Minkowski vacuum with vanishing fluxes into the AdS_4 vacuum with non-vanishing fluxes. The decay of the Minkowski vacuum occurs due to the creation of the domain wall, which sweeps through space-time until the entire universe is in the new AdS_4 vacuum. This is similar but not completely equal to the creation of a bubble via the Coleman/De Luccia instanton [149]. In fact in order to be realistic, one should break supersymmetry and uplift the Minkowski vacuum

by a small amount to obtain a de Sitter vacuum which decays into the AdS_4 vacuum. Neglecting the problem of supersymmetry breaking and the uplift, the decay amplitude of the Minkowski (de Sitter) vacuum is then given by the following expression:

$$\Gamma \simeq M_P \exp\left(-\frac{8\pi^2 M_P^4 C}{\sigma^2}\right) = M_P \exp\left(\frac{24\pi^2 M_P^4 C}{\Lambda_{\text{AdS}}}\right). \tag{4.125}$$

The constant C depends on the details of the domain wall solution.

As also discussed in [148], the corresponding decay amplitude is independent of the de Sitter cosmological constant $\Lambda = V_0$, but only depends on the value of Λ_{AdS}. In order to avoid too fast decay of our vacuum, $|\Lambda_{\text{AdS}}|$ must not be too large. e.g. if $|\Lambda_{\text{AdS}}| \simeq m_{3/2}^4$, the life-time of our universe is long enough. However AdS_4 vacua with $|V_1| \sim M_P^4$ create too much decay of our vacuum. Using the known expression for Λ_{AdS} in eq. (4.116), this constraint can be translated into the following restriction on the flux quantum numbers:

$$\frac{3^7 \sqrt{\frac{3}{5}}}{100} \frac{|\tilde{a}_0 \tilde{c}_1 \tilde{c}_2 \tilde{c}_3|(|\tilde{m}_0 \tilde{e}_1 \tilde{e}_2 \tilde{e}_3|)^{5/2}}{(\tilde{e}_1 \tilde{e}_2 \tilde{e}_3)^4} \ll 1. \tag{4.126}$$

5 String and brane inflation

5.1 General remarks

5.1.1 Inflation from scalar fields: Slow roll conditions — F- and D-term inflation

Recent astrophysical experiments have provided an enormous amount of high precision data about the early history of our universe. In fact, we know now that the universe is spatially flat, i.e. $\Omega = 1$, and the latest CMB data from WMAP5 [150] agree with an almost scale invariant spectrum with spectral scalar index:

$$n_s = 0.96 \pm 0.013. \tag{5.1}$$

(Note that this value assumes that there is basically no contribution from cosmic strings. In case cosmic strings contribute to the formation of large scale structures, the allowed value for n_s might be increased [151, 152], see also the later discussion.) Second there can be only small tensor perturbations, the relevant ration r of tensor to scalar perturbations is bound as follows:

$$r \equiv \frac{\Delta_T^2}{\Delta_\phi^2} < 0.30. \tag{5.2}$$

These limits will be further improved within the next years, most notably by the mission of the Planck satellite.

All these data can be nicely explained by an epoch of cosmic inflation in the early universe (for a nice text book on cosmology and inflation see [153]). Hence the goal will be to use the data from the CMB to find or to probe the fundamental theory of gravity and matter in the early universe. This strategy works best focussing on correlated signatures, e.g. on the correlation between n_s and the possible abundance of cosmic strings. The most

common method for this is to use effective scalar field theories wit a scalar potential $V(\phi)$, where ϕ is the inflaton field that drives inflation. Of course, a huge collection of effective field theories exist. So we would like to constrain the viable effective field theories as much as possible, first from the experimental data. Second, we also like to ask the question, which effective field theory can be consistently embedded into quantum gravity and into string theory. This will also give us some further constraints on inflationary effective theories, as we will discuss in the next subsection.

From observations we know that the potential $V(\phi)$ of inflation must be sufficiently flat. Specifically, the following slow roll conditions have to be satisfied:

$$\epsilon = -\frac{\dot{H}}{H^2} = \frac{M_P^2}{2}\left(\frac{V'(\phi)}{V(\phi)}\right)^2 \ll 1 \,,$$

$$\eta = \frac{\dot{\epsilon}}{\epsilon H} = M_P^2\left(\frac{V''(\phi)}{V(\phi)}\right)^2 \ll 1 \,. \tag{5.3}$$

The smallness of the two slow parameters follows, since the scalar spectral index is related to ϵ and η as

$$n_s = 1 - 6\epsilon + 2\eta \,. \tag{5.4}$$

Slow roll inflation can be roughly achieved in two different ways:

- Large field inflation: $\phi \geq M_P$.

  The prime example for large field inflation is chaotic inflation [154] with only a mass term in the inflaton potential:

$$V(\phi) = \frac{1}{2}m^2\phi^2 \,. \tag{5.5}$$

  Large field inflation generically leads to gravitational waves, i.e. to a large r parameter. Then the Lyth bound [155] relates the field range of ϕ to r,

$$r = 0.01\left(\frac{\Delta\phi}{M_P}\right)^2 \,, \tag{5.6}$$

  and hence observations tell us that $\Delta\phi \leq (5-6)M_P$.

- Small field inflation: $\phi \leq M_P$.

  Here there are no constraints from gravitational waves, but, as we will discuss, the scenario generically requires large fine tuning of parameters for eqs. (5.3) to be satisfied. In particular, the effective, F-term supergravity scalar potential (4.54) is not flat at all. So, assuming a canonical Kähler potential for the inflaton field, $K = \phi\bar{\phi}$, the effective supergravity potential generically behaves as

$$V \sim e^{|\phi|^2} \,. \tag{5.7}$$

As we will see, a possible way out is D-term inflation [156, 157] (hybrid inflation [158]) where the theory possesses a U(1) shift symmetry, such that the D-term scalar potential does not at all depend on the inflaton field, but a potential is only generated by Coleman-Weinberg loop effects.

5.1.2   Constraints from black hole decays

It is usually assumed that from the knowledge of low-energy perturbative physics (e.g., such as, the particle spectrum, and their couplings) in our vacuum, one cannot draw any conclusion about the physics in other vacua on the landscape, without knowing the non-perturbative structure of underlying high scale theory. This belief is based on the intuition, that different vacua correspond to different non-perturbative solutions of the high energy theory, largely separated by the expectation values of the classical order parameters (e.g., vacuum expectation values (VEVs) of the scalar fields), whereas low energy perturbative physics only accounts for small fluctuations about this solutions. As a result, even in the neighboring vacua, physics may be arbitrarily different and unpredictable for a low energy observer in our vacuum. We wish to show that black hole (BH) physics can provide a powerful guideline for overcoming this obstacle. Among, the expected enormity of the vacuum landscape, there is a large subset that shares common gravitational physics. In these vacua, the classical black hole physics is also common and imposes the same consistency constraints on perturbative particle physics.

In particular, by incorporating the consistency bounds, that BH physics imposes on number and masses of particle species [159], we can derive non-trivial constraints not only on our vacuum, but on any quasi-stationary state, which can be obtained by a continuous deformation of it. Under *continuous deformation*, we mean a change of expectation values that preserves invariant characteristics of the vacuum (such as, the number of species, their chirality, and possibly other topological characteristics). In a certain well-defined sense, to be made precise below, BH physics allows us to "see" through the landscape. In this part of the discussion, the key tool in our consideration will be a BH constraint on number of particle species and their masses. This bound can be derived from the flat space thought experiment, with BH formation and evaporation. In this experiment, an observer forms a classical BH and later detects its evaporation products. In each case, when the lifetime of a BH is less than the lifetime of the species, a powerful bound follows. For example, in the simplest case the number of stable species of mass M cannot exceed

$$N_{\max} \equiv \frac{M_P^2}{M^2} \, . \tag{5.8}$$

This consistency constraint must be satisfied in every vacuum of the theory. This fact automatically limits the number of possible deformations of our vacuum, which from perturbative physics alone one would never guess. For example, in our vacuum, *a priory*, we may have a very large number of massless species coupled to a modulus ϕ. Naively, nothing forbids existence of another vacuum, obtained by giving an arbitrary VEV to the modulus ϕ. However, since such a deformation of the vacuum gives masses to the species coupled to ϕ, only deformations permitted by the BH bound are possible. Thus, BH physics, automatically constraints physics in such vacua. The vacua in question does not have to be degenerate with ours, or even be stationary. Below we shall generalize BH bound for such vacua. Primary target of this study will be the de Sitter and quasi de Sitter vacua, that may be connected to ours by a continuous deformation of some scalar VEVs. The phenomenological importance of this study is obvious. Existence of such vacua is suggested

by the strong cosmological evidence that our Universe underwent a period of inflation, which is responsible for solving the flatness and the horizon problems, and creating the spectrum of density perturbations. Knowing that we, most likely, rolled down from another vacuum, we wish to understand constraints on such states by using BH physics, and whatever knowledge of perturbative physics we have in our present vacuum. The bounds from BH physics, which we discuss in this paper, set powerful criteria about what is the class of effective string actions, which can be consistently coupled to quantum gravity, and eventually capture string physics, which might have been lost in the effective action approach. Those effective field theories or vacua which cannot fulfill this criterion are called swampland [160] (see also [161]).

The generalization of the BH bound to the de Sitter and quasi de Sitter vacua relies on certain relations between the Schwarzschild radius and the lifetime of a "test" BH, and the Hubble radius and the lifetime of the corresponding (quasi) de Sitter vacuum respectively [162]. Shortly, for a given number and masses of species, there is an upper limit on the lifetime and the Hubble size of the vacuum, or else the BH bound (5.8) must be satisfied. In the other words, a given vacuum can only invalidate this BH bound on species, by becoming more curved and/or shorter lived. For the slow-roll inflationary vacua, this implies constraints on the slow-roll parameters, and subsequently, on the allowed number of the inflationary e-foldings.

We now generalize the BH proof of the bound to the de Sitter and inflation [162]. Let M be the mass of the species, and let H be the Hubble parameter in de Sitter. Consider a slow roll inflation driven by a single inflaton field ϕ. The equation for the spatially-homogeneous time-dependent field is,

$$\ddot{\phi} + 3H\,\dot{\phi} + V(\phi)' = 0\,, \tag{5.9}$$

where, prime stands for the derivative with respect to ϕ. The main idea of the slow roll inflation is, that for certain values of ϕ, the potential $V(\phi)$ is sufficiently flat, so that the friction term dominates and this allows ϕ to roll slowly. The energy density is then dominated by the slowly-changing potential energy. The Hubble parameter is approximately given by $H^2 \simeq V(\phi)/3M_P^2$, and can be regarded as constant on the time scales $\sim H^{-1}$. Obviously, the inflationary region of the potential must be away from todays minimum with almost zero vacuum energy. In any inflationary scenario the value of the inflaton field during inflation is very different from its todays expectation value ϕ_0 corresponding to the minimum of $V(\phi)$, which without loss of generality we can put at $\phi_0 = 0$.

Soon after the end of the inflationary period, inflaton oscillates about its true minimum ϕ_0, and reheats the Universe. For this to happen, inflaton should necessarily interact with the standard model particles and possibly with the other fields. Let us consider an inflaton coupled to N species, with masses M_j. For the efficient reheating, the masses of the the particles about the minimum ϕ_0, must be less than the inflaton mass about the same minimum. That is, $M_j \ll V''(\phi_0)$. Due to coupling to the inflaton field, the masses of species are functions of its expectation value, $M_j(\phi)$, and it is very common that these masses change substantially during inflation. The key point that we are willing to address

now, is that the masses of these species are subject to the BH bound, and give useful restriction on the inflationary trajectory. Thus, knowing the couplings of the inflaton in *our* vacuum, one can get an non-trivial information about the much remote inflationary vacua of the same theory.

For simplicity, we shall assume the universality of the species masses $M_j(\phi) = M(\phi)$. During the slow-roll inflation, Universe is in a quasi-de-Sitter state, in which the inflationary Hubble parameter sets the size of the causally-connected event horizon H^{-1}. However, the difference from the stationary de Sitter vacua, is that in realistic inflationary scenarios the slow roll phase (in any given region) is not exponentially long lived, and lasts for several Hubble times. So H^{-1} sets the time scale on which parameters can be regarded as constant.

Thus, a hypothetical observer located within a given causally-connected inflationary patch can perform a sensible experiment with BH formation and evaporation. In such a case, the black hole bound can be directly applied, and we arrive to the bound [162]

$$M(\phi) < \frac{M_P}{(H^{-1}(\phi)M_P)^{\frac{1}{3}}}. \tag{5.10}$$

All the information that this bound implies for a given inflationary scenario, is encoded in the functions $M(\phi)$ and $H(\phi)$. We shall now illustrate this on some well known examples.

5.2 Several scenarios

5.2.1 Chaotic inflation

Let us consider the example of Linde's chaotic inflation [154]. This is based on a single scalar field with a mass m and no self-coupling

$$V(\phi) = \frac{1}{2}m^2\phi^2 + g\phi\bar{\psi}_j\psi_j. \tag{5.11}$$

The last term describes the coupling to N-species, which for definiteness we assume to be fermions, and g is the interaction constant. As said above, the coupling of the inflaton to the species is crucial for the reheating.

The above theory has a Minkowski vacuum, in which $\phi = 0$ and all the species are massless. Due to the latter fact, in this vacuum the BH bound on the number and mass of the species is satisfied. However, as we shall see, the same bound, puts non-trivial restriction on the inflationary epoch, since during inflation $\phi \neq 0$ and species are massive.

Ignoring for a moment the coupling to the species, the logic in the standard Chaotic inflationary scenario goes as follows. The expectation value of the field ϕ can be arbitrarily large, as long as the energy density remains sub-Planckian, that is

$$m^2\phi^2 \ll M_P^4. \tag{5.12}$$

The equation (5.9) then can be applied and takes the form

$$\ddot{\phi} + 3H\dot{\phi} + m^2\phi = 0, \tag{5.13}$$

where $H^2 = \frac{m^2\phi^2 + \dot{\phi}^2}{6M_P^2}$. As long as $H \gg m$, the friction dominates and ϕ rolls slowly. This implies (up to a factor of order one)

$$\phi \gg M_P, \tag{5.14}$$

which is compatible with (5.12) as long as $m \ll M_P$. If the above is satisfied, ϕ rolls slowly, and Universe undergoes the exponentially fast expansion. Let us now see how the coupling to the species restricts the above dynamics. During inflation the mass of the species is $M = g\phi$ and they are subject to the BH bound. To see what this bound implies we can simply insert the current values of $M(\phi)$ and $V(\phi)$ in (5.10), and we get

$$g\phi \lesssim M_P \left(\frac{m\phi}{M_P^2} \right)^{\frac{1}{3}}. \tag{5.15}$$

Non-triviality of the above constraint is obvious. For example, the standard argument assumes that inflation could take place for arbitrary $m \ll M_P$, and from arbitrarily large values of ϕ satisfying (4.94), irrespective to the number of species to which inflaton is coupled. The above expression tells us that in the presence of species, this is only possible, provided, $g \lesssim (M_P/\phi)^{2/3}(m/M_P)^{1/3}$.

For the practical reasons of solving the flatness and the horizon problems, in the standard Chaotic scenario, last 60 e-foldings happen for $\phi \lesssim 10M_P$, whereas from density perturbation we have $m \sim 10^{12}$GeV or so. This implies, $g < 10^{-3}$. This constraint can be easily accommodated by the adjustment of couplings, however it is remarkable that no fine tuning can make $g \sim 1$ consistent.

5.2.2 Hybrid inflationary vacua

The essence of the hybrid inflation [158] is that inflationary energy density is not dominated by the potential of the slowly-rolling inflaton field ϕ, but rather by a false vacuum energy of other scalar fields, χ_j. These fields are trapped in a temporary minimum, created due to large positive mass²-s, which they acquire from the coupling to the inflaton field. The slowly rolling inflaton then acts as a clock, which at some critical point triggers the transition that liberates the trapped fields, and converts their false vacuum energy into radiation. However, usually Inflation ends before this transition, because of breakdown of the slow-roll. Thus, in hybrid inflation, the presence of fields with inflaton-dependent masses is essential not only for the reheating, but for the inflation itself. The simplest prototype model realizing this idea is

$$V = \lambda^2 \phi^2 \chi_j^2 + \left(\frac{g}{2} \chi_j^2 - \mu^2 \right)^2, \tag{5.16}$$

where λ and g are constants. Then, for $|\phi| > \phi_t \equiv \mu\sqrt{\frac{g}{\lambda^2}}$, the effective potential for χ_j is minimized at $\chi_j = 0$, and the false vacuum energy density is a ϕ-independent constant, μ^4. Thus, in the classical treatment of the problem, starting at arbitrary initial value $\phi \gg \phi_t$ and with zero initial velocity, ϕ would experience zero driving force and system would inflate forever. One could slightly lift this flat direction by adding an appropriate
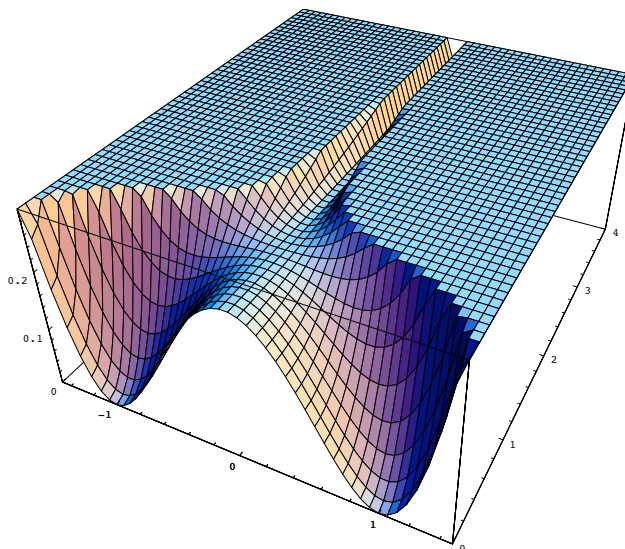
Figure 15. The potential for hybrid inflation as a function of the inflaton ϕ (in the y-direction)and the tachyon field χ (in the x-direction).

self interaction potential for ϕ (e.g., such as a positive mass term $m^2\phi^2$) which would drive ϕ towards the small values. In such a picture inflation ends abruptly after ϕ drops to its critical value ϕ_t, for which χ_j becomes tachyonic, and system rapidly relaxes into the true vacuum. This is shown in figure 15. However, the above story is only true classically, and quantum mechanical corrections are very important and always generate potential for ϕ. Because of to these corrections, typically, inflation ends way before the phase transition, due to breakdown of the slow-roll. Existence of supersymmetry cannot change the latter fact, however, supersymmetry does make the corrections to the potential finite and predictive.

The simple supersymmetric realizations of the hybrid inflation idea have been suggested in form of F-term [163, 164] and D-term [156, 157] inflationary models. As a result of supersymmetry, in F-term inflation $\lambda = g$. Due to renormalization of the Kähler function via χ_j loops, the non-trivial inflaton potential is inevitably generated, which for $\phi \gg \phi_t$ has the following Coleman-Weinberg form

$$V(\phi) \simeq \mu^4 \left[1 + \frac{Ng^2}{16\pi^2}\ln\frac{g|\phi|}{Q}\right],\tag{5.17}$$

where, Q is the renormalization scale. Notice, that this potential cannot be fine tuned away by addition of some local counter terms. The condition of the slow roll is that $V'' \ll H^2$, implying that

$$N\,g^2 \ll \frac{\phi^2}{M_P^2}\,.\tag{5.18}$$

Because of the logarithmic nature, the slope flattens out for large ϕ. However, even if one tries to ignore any other correction to the potential, nevertheless, the slow-roll condition will eventually run in conflict with the black hole bound, which implies that

$$N\,g^2 \lesssim \frac{M_P^2}{\phi^2}\,.\tag{5.19}$$

This fact indicates, that even if the theory is in seemingly-valid perturbative regime (that is, $\frac{Ng^2}{16\pi^2}\ln\frac{g\phi}{Q} \ll 1$), nevertheless, the perturbative corrections to the Kähler cannot be the whole story, and theory has to prevent growth of ϕ, by consistency with the black hole physics.

5.3 String inflationary vacua

String theory provides an interesting and also promising microscopic framework to realize cosmic inflation (for reviews on string inflation and string cosmology see [165–169]). The 4D effective potential for the inflaton typically arises after compactification from ten to four space-time dimensions. Specifically consider the following effective potential as a function of the moduli scalar fields M:

$$V(M) = V_0(M') + V'(M',\phi) \quad \implies \quad V(\phi) = V_0 + V'(\phi). \tag{5.20}$$

here we have assumed that we can slit the moduli fields M into one inflaton field ϕ and the remaining fields M'. The potential $V(M')$ can be very steep and fixed all moduli fields M' to some particular values, where we have to assume that the minimum V_0 of this part of the potential is positive, i.e. it forms a de Sitter minimum. The remaining potential after stabilization of all M'-fields is then the inflaton potential, which has to meet the slow roll conditions discussed before. However often, the typical string compactification does not give rise to vanilla potentials of this type, as one can in general not separate out a stabilizing potential $V_0(M')$ that does not depend on the putative inflaton ϕ. Instead one usually has moduli stabilization potentials $V_0(M',\phi)$ that also depend on ϕ and hence generically interfere with slow-roll inflation. Another potential problem, which will be the most relevant for the present paper, is that the value V_0 at which all orthogonal moduli M' can be stabilized might be constrained to be negative due to the peculiar structure of the scalar potential $V_0(M')$. Then one might always have at least one steep direction whenever the potential is positive. Potentially dangerous moduli of this type are, in particular, the overall volume modulus of a compactification manifold or the dilaton, as these often enter as steep directions in many contributions to the scalar potential.

Now we can imagine two scenarios for string inflation:

- Closed string inflation: the inflaton is a closed string modulus.

  Here the effective potential is due to the potential energy from fluxes, including geometrical fluxes, and/or (Euclidean) branes wrapped around internal cycles.

- Open string inflation: inflaton is a open string modulus, the distance between D-branes.

  Here the effective potential is due to the attractive or repulsive forces between (non-BPS) branes.

Let us discuss the mechanism of brane inflation [170, 171] in more detail. In this picture the role of the inflaton field ϕ is played by the brane-separation field. A simplifying but crucial assumption of the original brane inflation model, is that compactification moduli

are all fixed, with the masses being at least of order of the inflationary Hubble parameter, so that branes can be considered to be moving in a fixed external geometry, weakly affected by the brane motion. In the same time, the $4d$ Hubble volume must be larger than the size of the compact extra dimensions. These conditions allow us to apply the power of the effective four-dimensional supergravity reasoning. Below we shall focus on the case of D-brane inflation, based on the motion and subsequent annihilation of branes an anti-branes. This picture from the four-dimensional perspective can be understood as the hybrid inflation, in which ϕ is a brane distance field, and role of χ is played by the open string tachyon. In this picture, the supersymmetry breaking by a non-BPS brane-anti-brane system corresponds to the spontaneous supersymmetry breaking via FI D-term.

When branes are far apart, there is a light field ϕ, corresponding to their relative motion. This mode is a combination of the lowest lying scalar modes of the open strings that are attached to a brane or anti-brane only. We are interested in the combination that corresponds to the relative radial motion of branes.

$$\phi = M_{\mathrm{string}}^2 r \,. \tag{5.21}$$

In the simplest case of a single brane-anti-brane pair, we have the two gauged U(1)-symmetries. One of these two provides a non-vanishing D-term. The tachyon (χ) is an open string state that connects the brane and the anti-brane. The mass of this stretched open string is $M_s^2 r$. In $4d$ language, the tachyon as well as other open string states get mass from the coupling to ϕ.

The energy of the system is given by the D-term energy, which is constant at the tree-level, but not at one-loop level. At one-loop level the gauge coupling depends on ϕ. g^2 gets renormalized, because of the loops of the heavy U(1)-charged states, with ϕ-dependent masses. For instance, there are one-loop contributions from the χ and $\bar{\chi}$ loops. More precisely there is a renormalization of g^2 due to one-loop open string diagram, which are stretched between the brane and anti-brane. Since the mass of these strings depend on ϕ, so does the renormalized D-term energy

$$V_D = \frac{g^2(\phi)}{2} D^2 = \frac{g_0^2}{2} \left(1 + g_0^2 f(\phi) \right) \xi^2 \,, \tag{5.22}$$

where g_0^2 is the tree-level gauge coupling, and $f(\phi)$ is the renormalization function. For example, for D_3-D_7 system, discussed in section (5.3.2) at the intermediate distances $(M_{\mathrm{string}}^{-1} \ll r \ll R$, where R is the size of two transverse extra dimensions), this takes the form (5.17).

We shall now see, why at least in the simplest D-brane setup, the U(1) symmetry must be Higgsed throughout the inflation. Let us again think about the process of D_{3+q}-$\bar{D}_{3+q}$ driven inflation, with the subsequent brane annihilation. We assume that q dimensions are wrapped on a compact cycle, and relative motion takes place in $6-q$ remaining transverse dimensions.

The low energy gauge symmetry group is U(1)$\times$U(1), one linear superposition of which is Higgsed by the tachyon VEV. The crucial point is, that this Higgsed U(1) gauge field is precisely the combination of the original U(1)'s that carries a non-zero RR-charge (the

other combination is neutral). The corresponding gauge field strength $F_{(2)}$ has a coupling to the closed string RR $2 + q$-form $C_{(2+q)}$ via the WZ terms,

$$\int_{3+1+q} F_{(2)} \wedge C_{(2+q)} \,, \tag{5.23}$$

where, since we are interested in the effective $4d$ supergravity description, we have to integrate over extra q-coordinates, and only keep the $4d$ zero mode component of the RR field. This then becomes an effective 2-form, $C_{(2)}$.

The connection with the $4d$ supergravity D-term language, is made by a dual description of the $C_{(2)}$-form in terms of an axion (a),

$$d\mathrm{C}_{(2)} \rightarrow *d\mathrm{a}\,, \tag{5.24}$$

where star denotes a $4d$ Hodge-dual. Under this duality transformation we have to replace

$$(d\mathrm{C}_{(2)})^2 \,+\, \frac{\xi}{M_P^2} F_{(2)} \wedge \mathrm{C}_{(2)} \quad \rightarrow \quad M_P^2 (d\mathrm{a} \,-\, \mathrm{gQ_a W})^2 \,, \tag{5.25}$$

where $Q_a = \frac{\xi}{M_P^2}$ is the axion charge under U(1). As it should, this charge vanishes as the compactification volume goes to infinity, and $4d$ supergravity approaches the rigid limit. We thus see that the U(1) gauge field (W_μ) acquires a mass $m_W^2 \gtrsim \xi^2 / M_P^2$.

The D-term (hybrid) leads to a very specific prediction, namely to the appearance of cosmic strings as non-trivial topological defects after the spontaneous breaking of the U(1) $\times$ U(1) gauge symmetry [172]. Since cosmic strings contribute to the energy density of the universe, to the generation of the CMB and can be also visible by cosmic lensing, there are stringent experimental bounds on the abundance of cosmic strings in the universe. These bounds are pretty dangerous for open string D-term inflation, resp. can be used to give strong constraints on these kind of models, as we will further discuss in section (5.3).

5.3.1 The search for type IIA inflation and de Sitter vacua with positive cosmological constant

In the section (4.4), we have derived part of the low energy effective action for the type IIA, AdS_4 compactifications. Here we would briefly like to discuss the question if some of these IIA vacua can be used for inflation [78, 173]. Specifically, we want to address the question if the scalar potential in the closed string moduli sector can be flat enough in order to allow inflation by one of the closed string moduli. Therefore the parameter ϵ must be small enough in some region of the closed string scalar potential. In addition, this analysis is also relevant for open string inflation on these IIA vacua, since in this case we have to find closed string minima of the scalar potential, i.e. $\epsilon = 0$ somewhere in the closed string moduli space. Extending the earlier work [174], the authors of [175] proved a no-go theorem against small ϵ, i.e. against a period of slow-roll inflation in type IIA compactifications on Calabi-Yau manifolds with standard RR and NSNS-fluxes, D6-branes and O6-planes at large volume and with small string coupling. More precisely, they show that the slow-roll parameter ϵ is at least $\frac{27}{13}$ whenever the potential is positive, ruling out slow-roll inflation in a near-de Sitter regime, as well as meta-stable dS vacua. As emphasized in [175], however, the

inclusion of other ingredients such as NS5-branes, geometric fluxes and/or non-geometric fluxes evade the assumptions that underly this no-go theorem. In fact in [176] de Sitter vacua in type IIA were found using some of these additional ingredients. Furthermore a concrete string inflationary model on nilmanifolds with D4-branes and large field inflation was presented in [177, 178]. The coset models of section (4.4) could thus be candidates for circumventing the no-go theorem as they all have geometric fluxes. So let us study this in some more detail.

The proof of this no-go theorem is remarkably simple and uses only the scaling properties of the scalar potential with respect to the volume modulus

$$\rho = \left(\frac{\text{Vol}}{\text{vol}}\right)^{1/3},$$ (5.26)

where $\text{vol} = |\int e^{123456}|$ is a standard volume, and the dilaton modulus

$$\tau = e^{-\Phi}\sqrt{\text{vol}},$$ (5.27)

as well as the signs of the various contributions to the potential.

Classically, the four-dimensional scalar potentials of such compactifications may receive contributions from the NSNS H_3-flux, geometric fluxes f^i_{jk}, O6/D6-branes and the RR-fluxes F_p, $p = 0, 2, 4, 6$:

$$V = V_3 + V_f + V_{O6/D6} + V_0 + V_2 + V_4 + V_6,$$ (5.28)

where $V_3, V_0, V_2, V_4, V_6 \geq 0$, and V_f and $V_{O6/D6}$ can a priori have either sign.

In [175] the authors studied the dependence of this scalar potential on the volume modulus $\rho = (\text{Vol})^{1/3}$ and the four-dimensional dilaton $\tau = e^{-\phi}\sqrt{\text{Vol}}$. Using only this (ρ, τ)-dependence, they could derive a no-go theorem in the absence of metric fluxes that puts a lower bound on the first slow-roll parameter,

$$\epsilon \equiv \frac{K^{A\bar{B}}\partial_A V \partial_{\bar{B}} V}{V^2} \geq \frac{27}{13}, \quad \text{whenever } V > 0,$$ (5.29)

where $K^{A\bar{B}}$ denotes the inverse Kähler metric, and the indices $A, B, \ldots$ run over all moduli fields. This then not only excludes slow-roll inflation but also de Sitter vacua (corresponding to $\epsilon = 0$).

The lower bound (5.29) follows from the observation that a linear combination of the derivatives with respect to ρ and τ is always greater than a certain positive multiple of the scalar potential V. More precisely, the general scalings

$$V_3 \propto \rho^{-3}\tau^{-2}, \qquad V_p \propto \rho^{3-p}\tau^{-4}, \qquad V_{O6/D6} \propto \tau^{-3}, \qquad V_f \propto \rho^{-1}\tau^{-2}$$ (5.30)

imply, for the scalar potential (5.28),

$$-\rho\frac{\partial V}{\partial \rho} - 3\tau\frac{\partial V}{\partial \tau} = 9V + \sum_{p \,\epsilon\, 2,4,6} pV_p - 2V_f.$$ (5.31)

Hence, whenever the contribution from the metric fluxes is zero or negative, the right hand side in (5.31) is at least equal to $9V$, which can then be translated to the above-mentioned lower bound $\epsilon \geq \frac{27}{13}$ [175]. Avoiding this no-go theorem without introducing any new ingredients would thus require $V_f > 0$. Since $V_f \propto -R$, where R denotes the internal curvature scalar, this is equivalent to demanding that the internal space have negative curvature. Since all terms in V scale with a negative power of τ we see from (5.28) and (5.30) that we would also need $V_{O6/D6} < 0$ to avoid a runaway.

In summary, if geometric fluxes *alone* are to circumvent this no-go theorem, they can do so at most if they are positive:

$$V_f > 0 \qquad \text{(Necessary condition for evading the no-go theorem)}. \qquad (5.32)$$

In fact, we can immediately find the geometric part of the potential from the Einstein-Hilbert term in the ten-dimensional action:

$$V_f = -\frac{1}{2} M_P^4 \kappa_{10}^2 e^{2\Phi} \text{Vol}^{-1} \text{R} = -\frac{1}{2} \text{M}_{\text{P}}^4 \kappa_{10}^2 \tau^{-2} \text{R} \,, \qquad (5.33)$$

where R is the scalar curvature of the internal manifold. For cosets/group manifolds R can be explicitly calculated. This expression has indeed the expected scaling behavior since $R \propto g^{-1} \propto \rho^{-1}$. It follows that the condition (5.32) for avoiding the no-go theorem can be rephrased as

$$R < 0 \,. \qquad (5.34)$$

Let us display the scalar curvature for some of our coset models in section 4.4:

$$
\begin{aligned}
\frac{\text{G}_2}{\text{SU}(3)} : & \qquad R = \frac{10}{k_1} \,, \\
\frac{\text{Sp}(2)}{\text{S}(\text{U}(2) \times \text{U}(1))} : & \qquad R = \frac{6}{k_1} + \frac{2}{k_2} - \frac{k_2}{2(k_1)^2} \,, \\
\frac{\text{SU}(3)}{\text{U}(1) \times \text{U}(1)} : & \qquad R = 3\left(\frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}\right) - \frac{1}{2}\left(\frac{k_1}{k_2 k_3} + \frac{k_2}{k_1 k_3} + \frac{k_3}{k_1 k_2}\right) \,, \\
\frac{\text{SU}(3) \times \text{U}(1)}{\text{SU}(2)} : & \qquad R = \frac{1}{\sqrt{1+\rho^2}}\left(\frac{6}{k_1} - \frac{3\rho k_2}{4(1+\rho^2)k_1^2}\left|\frac{U_2}{U_1}\right|\right) \,,
\end{aligned}
\qquad (5.35)
$$

where $k_i > 0$ are the Kähler moduli and U_i the complex structure moduli that enter the expansion of J and $\text{Im}\Omega$. We see that for $\frac{\text{G}_2}{\text{SU}(3)}$ the curvature is always positive, so inflation is still excluded, however for the other models there are values of the moduli such that $R < 0$. For SU(2)×SU(2) we did not display the curvature, because taking generic values of the complex structure and Kähler moduli, its expression is quite complicated and not very enlightening. However, also in that case it is possible to choose the moduli such that $R < 0$.

Note that this does not yet guarantee that the ϵ parameter is indeed small, it just says that the theorem that requires it to be at least 27/13 no longer applies. Hence, a logical next step would be to calculate ϵ in this region, ideally by taking also all other moduli into account (see the general expression and try to make ϵ small or zero. These would be necessary conditions for, respectively, inflation or de Sitter vacua. They are not sufficient

however, because for inflation, we would also need the η parameter to be small and further obtain a satisfactory inflationary model which could end in a meta-stable de Sitter vacuum etc. For a meta-stable de Sitter vacuum, on the other hand, one would also have to check that the matrix of second derivatives only has negative eigenvalues.

5.3.2   Type IIB D3/D7-brane inflation

Except the large field inflation model in type IIA on nilmanifolds of [177] most of the inflationary string inflationary models are so far in within type IIB compactifications. In particular, a well studied case is given in terms of warped inflation on a IIB Calabi-Yau, with D3($\overline{\text{D3}}$)-branes located at the tip of a long throat in the internal geometry [179–186]. This class of models realizes F-term inflation with relatively small inflaton field, being the distance between D3- and $\overline{\text{D3}}$ brane on the throat. Considerable fine tuning of parameters is necessary in warped inflation in order to meet the constraints of slow roll inflation. Here we like to discuss another working example of type IIB inflation, namely the IIB orientifold on $K3 \times T^2/\mathbb{Z}_2$ with D3- and D7-branes [187–192]. As we will see this model provides a concrete realization of D-term inflation with a U(1) shift symmetry, which ensures the flatness of the potential. However quantum corrections destroy the shift symmetry and also generate a F-term contribution to the effective potential. So again some amount of fine-tuning is required. As we will discuss it is nevertheless possible to directly confront this model with experimental data, which makes the model interesting in itself.

As said already, the model we would like to study here is D3/D7-brane inflation on the background $K3 \times T^2/\mathbb{Z}_2$. The resulting model is a stringy version of a hybrid D-term inflation model discussed above [156, 172, 193, 194] with a waterfall stage at the end in which a charged scalar field condenses.[9] As a D-term inflation model, D3/D7-brane inflation, a priori, does not suffer from the generic supergravity eta-problem of F-term inflation models. The main problem of D-term inflation is instead the cosmic string production during the waterfall stage, when the spontaneous breaking of the underlying U(1)-symmetry takes place and the D-flatness condition is restored.

One of the reasons to study the D3/D7-model on $K3 \times T^2/\mathbb{Z}_2$ is its high computability. Type IIB string theory compactified on $K3 \times T^2/\mathbb{Z}_2$ is related to M-theory compactified on $K3 \times K3$ [128, 195, 196] and is associated with 4D, $\mathcal{N} = 2$ supergravity specifically described in [197–199]. Bulk moduli stabilization in these models was studied in a series of papers, and it is one of the best understood string theory models with stabilization of all bulk moduli. In its simplest incarnations this model does not contain the D-branes necessary to describe the Standard Model of particle physics at low energies. Therefore, the D3/D7-system studied here should be regarded as a brane/flux module, which is responsible for inflation and moduli stabilization, and which has to be complemented by additional D-branes in order to obtain realistic Standard Model phenomenology at lower energies.

---

[9]This condensing field corresponds to a particular state of the strings stretching between the D3- and D7-brane, which becomes tachyonic at a certain critical interbrane distance due to the world volume flux on the D7-brane. The D3-brane is then dissolved on the D7-brane as an instanton, and $\mathcal{N} = 1$ supersymmetry becomes restored [188].

In the D3/D7-brane inflationary model, an attraction between a D3- and a D7-brane is triggered by a non-self-dual world volume flux on a D7-brane, which we will henceforth call the Fayet-Iliopoulos (FI) D7-brane. If both branes are space-time-filling, and the D7-brane wraps the $K3$-factor, the transverse interbrane distance on $T^2/\mathbb{Z}_2$ plays the role of the inflaton. A distinguishing feature of this model (as compared, e.g., with $D3/\overline{D3}$-brane inflation) is that the supersymmetry breaking during the slow-roll de Sitter phase is spontaneous, and hence well-controlled. More precisely, the supersymmetry breaking can be understood in terms of a two-step process: Certain *bulk* three-form fluxes on $K3 \times T^2/\mathbb{Z}_2$ may spontaneously break the original $\mathcal{N} = 2$ supersymmetry preserved by the geometry to $\mathcal{N} = 1$. In the resulting effective $\mathcal{N} = 1$ theory, the *world volume* fluxes on the D7-brane then give rise to a D-term potential. Assuming the volume modulus of the K3-factor to be fixed, this D-term potential is non-zero for sufficiently large D3-D7-distance, breaking supersymmetry spontaneously to $\mathcal{N} = 0$. This final spontaneous supersymmetry breaking induces a Coleman-Weinberg type one-loop correction to the scalar potential that drives the D3-brane towards the FI D7-brane. This motion corresponds to the phase of slow-roll inflation.

The effective Kähler potential. The NSNS- and RR-two-forms with one leg along the non-compact directions and one along the torus give rise to four vector fields in 4D. One linear combination of these four vectors corresponds to the 4D graviphoton, whereas the other three enter three vector multiplets. The three complex scalars of these vector multiplets are[10]

$$s = C_{(4)} - i\text{Vol}(K3) \,, \tag{5.36}$$

$$t = \frac{g_{12}}{g_{11}} + i\frac{\sqrt{\det g}}{g_{11}} \,, \tag{5.37}$$

$$u = C_{(0)} - ie^{\varphi} \tag{5.38}$$

which denote, respectively, the $K3$-volume modulus with its axionic RR-partner $C_{(4)}$, the T^2 complex structure modulus and the axion-dilaton.

The position moduli of the 16 D7-branes on the torus are denoted by y_7^k ($k = 1, \ldots, 16$). Depending on where one chooses the origin of these coordinates, they could obviously be defined in various ways. A very convenient way to define them for brane configurations close to the orientifold limit is to use $y_7^{1,2,3,4}$ for the complex positions of branes number 1-4 with respect to fixed point number 1, and similarly, to use $y_7^{5,6,7,8}$ to denote the positions of the branes number 5-8 with respect to fixed point number 2, and so forth. In this notation, $y_7^k = 0$ for all $k = 1, \ldots, 16$ thus would mean that there are four D7-branes sitting on top of each O7-plane, and we are at the orientifold limit with constant dilaton.

Classically, the moduli space of the vector multiplet sector is described by the special Kähler manifold

$$\mathcal{M}_V \cong \left(\frac{\text{SU}(1,1)}{\text{U}(1)}\right)_s \times \frac{\text{SO}(2,18)}{\text{SO}(2) \times \text{SO}(18)} \,, \tag{5.39}$$

---

[10]Unlike in section 4.2. we are using the standard notation of [197] for the moduli of $K3 \times T^2$.

where the first factor is parametrized by s, and the remaining scalars (t, u, y_7^k) span the second factor. This geometry can be obtained from the following cubic prepotential:

$$\mathcal{F}(s, t, u, y_7^k) = stu - \frac{1}{2} s y_7^k y_7^k \ . \tag{5.40}$$

In F-theory language, the dilaton, u, corresponds to the complex structure of the elliptic fiber of a second $K3$ factor, which we will denote by $\widetilde{K3}$. In this picture, the $\mathrm{SO}(2, 18)/(\mathrm{SO}(2) \times \mathrm{SO}(18))$ factor of $\mathcal{M}_V$ describes the complex structure moduli space of $\widetilde{K3}$. It should be noted that, far away from the orientifold limit, the convenient separation of the scalars into closed and open string moduli is in general no longer possible, and the 10D meaning of e.g. y_7^k as brane positions is less clear [128].

$$K = -\ln\Big[-8\,(\mathrm{Im}(s)\,\mathrm{Im}(t)\mathrm{Im}(u) - \frac{1}{2}\,\mathrm{Im}(s)\,(\mathrm{Im}(y_7^k))^2 - \frac{1}{2}\,\mathrm{Im}(u)\,(\mathrm{Im}(y_3^r))^2)\Big] \ . \tag{5.41}$$

The remaining moduli of the original $K3$-factor, as well as the torus volume and the remaining axions from the RR-four-form with two legs along K3 and two legs along $T^2/\mathbb{Z}_2$ live in altogether 20 hypermultiplets and parametrize, at tree-level, the quaternionic Kähler manifold

$$\mathcal{M}_H = \mathrm{SO}(4, 20)/(\mathrm{SO}(4) \times \mathrm{SO}(20)) \ . \tag{5.42}$$

This manifold has 22 translational isometries along the 22 real axionic directions, C^I $(I = 1, \ldots, 22)$, which descend in the above-mentioned way from the RR-four-form. These 22 axions transform in the vector representation of $\mathrm{SO}(3, 19) \subset \mathrm{SO}(4, 20)$, and hence decompose into an $\mathrm{SO}(3)$ triplet C^m $(m = 1, 2, 3)$ and an $\mathrm{SO}(19)$-vector C^a $(a = 1, \ldots, 19)$.

Volume stabilization due fluxes and due to a non-perturbative superpotential.
Three-form fluxes on $K3 \times T^2/\mathbb{Z}_2$ lead to a superpotential of the form

$$W_{\text{flux}} = W_H + W_F = \int_{K3 \times T^2} \Omega \wedge (F_3 + uH_3) \ , \tag{5.43}$$

where H_3 and F_3 denote the NSNS and RR three-form field strengths, respectively. These fluxes generically stabilize the moduli (t, u, y_7^r) and may lead to spontaneous partial supersymmetry breaking $\mathcal{N} = 2 \to \mathcal{N} = 1$. In an $\mathcal{N} = 1$ vacuum, one of the two $\mathcal{N} = 2$ gravitini (together with some of the other fields) gains a mass. In addition the fluxes have to satisfy the tadpole condition

$$\frac{1}{2}N_{\text{flux}} + N_{D3} = 24 \ , \tag{5.44}$$

where

$$N_{\text{flux}} = \frac{1}{(2\pi)^4(\alpha')^2} \int_{K3 \times T^2} H_3 \wedge F_3 \tag{5.45}$$

with the integral being evaluated on the covering torus (which explains the factor of $1/2$ in front of N_{flux} in (5.44)).

Volume stabilization of the K3 space is finally is achieved by a non-perturbative F-term potential due to either Euclidean D3-brane instantons or gaugino condensation on stacks of D7-branes, which may arise after spontaneous breaking of supersymmetry to $\mathcal{N} = 1$.

Here, we only focus on the volume of the $K3$-factor (the other Kähler moduli could be stabilized by Euclidean D3-brane instantons [196][11]). Moreover, we restrict ourselves to the mechanism of gaugino condensation. This implies a constraint on the charged matter spectrum of the brane setup, which has to allow for the presence of a non-perturbative superpotential from gaugino condensation (for the case of Euclidean D3-branes analogous constraints were discussed in [120, 196, 200]).

Thus, in order to comply with our notation from eq. (5.36) and, we define the $\mathcal{N} = 1$ gauge kinetic function f_{D7} as

$$f_{D7} = is \tag{5.46}$$

so that the gauge coupling is given by the real part of f_{D7}.

In order to ensure the appearance of a non-perturbative superpotential one has to require that the quantity

$$c = \sum_j T(r_j) - T(\text{adj}) \tag{5.47}$$

be negative. In (5.47), the sum runs over the light (charged) $\mathcal{N} = 1$ chiral multiplets in the representation r_j of the gauge group. In particular, no adjoint matter is allowed in the light spectrum of the $\mathcal{N} = 1$ gauge theory. We assume that the charged matter content of the D7-brane gauge theory is such that it fulfills $c < 0$, for example by giving mass to unwanted matter via fluxes [128, 200, 201].

$$W = W_{\text{flux}} + A_0 \exp\left(\frac{8\pi^2 f}{c}\right) = W_{\text{flux}} + A_0 e^{\frac{i8s\pi^2}{c}}, \tag{5.48}$$

The D7-brane stack on which gaugino condensation takes place should be at a different position on $T^2/\mathbb{Z}_2$ than the D7-brane on which world-volume flux is supposed to attract the D3-brane (see, e.g., figure 16 for a possible realization). Otherwise, the $K3$-volume is destabilized after inflation. In our model, the role of the attracting anti-D3-brane is played by the D7-brane with the world-volume flux on it, i.e., by the FI D7 brane. It should thus likewise be placed away from the stack of the volume stabilizing D7's so as to avoid the destabilization of the volume at the exit from inflation.

Inflationary D-term potential. In our model, the role of the attracting anti-D3-brane is played by the D7-brane with the world-volume flux on it, i.e., by the FI D7 brane. It should thus likewise be placed away from the stack of the volume stabilizing D7's so as to avoid the destabilization of the volume at the exit from inflation. The inflaton potential is then generated by spontaneous supersymmetry breaking due to a non-selfdual world volume flux on another D7-brane, which then triggers an attraction of a nearby D3-brane towards that D7-brane. In 4D, the supersymmetry breaking due to the world-volume fluxes can be attributed to a non-vanishing D-term potential. It is important that the D7-brane with the world volume flux is different from the D7-branes on which gaugino condensation takes

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[11]These Euclidean D3-instantons necessarily wrap the $T^2/\mathbb{Z}_2$-factor. As the only open string dependence of the resulting superpotentials is via the transverse distance between the space-time filling D3-brane and the corresponding D3-instanton, these superpotentials are independent of the D3-brane position along $T^2/\mathbb{Z}_2$ and, hence, the inflaton.
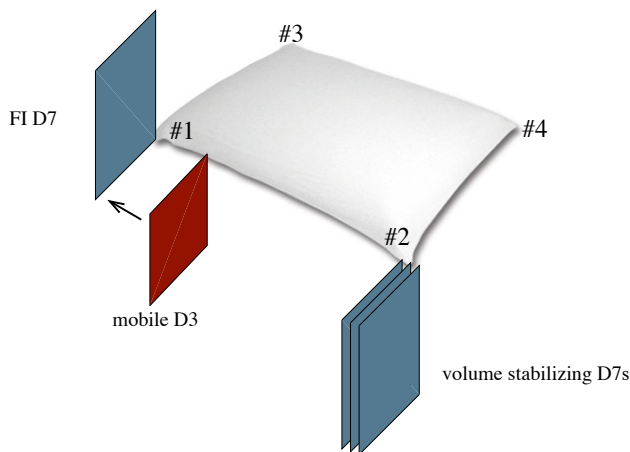
Figure 16. Brane realization for D3/D7-inflation. The K_3 volume is stabilized by a stack of D7-branes at fixed point No. 2; supersymmetry breaking by the the world volume flux occurs at the D7-brane at fixes point No. 1, and the D-term, inflaton potential is generated by the force between the mobile D3-brane and the FI D7-brane.

place and that both types of D7-branes are at different locations on $T^2/\mathbb{Z}_2$. The reason for this is that the function $A(y_3, \ldots)$ (see next section) entering the non-perturbative superpotential (5.48) vanishes if the D3-brane sits on top of the D7-branes responsible for the gaugino condensation [202]. If the gaugino condensation D7-branes and those with world-volume flux were the same, this would lead to volume destabilization at the end of inflation, when the D3-brane dissolves as an instanton on the D7-branes. The situation is thus similar to the setup described in [181, 183], where the mobile D3-brane also moves away from the volume stabilizing D7-branes and approaches the anti-D3-brane at the tip of the throat. The analogue of the anti-D3-brane would then be the D7-brane with world volume flux in our setup.

The $\mathcal{N} = 2$ theory with prepotential (5.40) features a shift symmetry for the Kähler potential and the D7-brane gauge kinetic function along the real parts of the D3-brane position moduli, y_3^r. If we assume that the D7-brane with the non-self-dual world-volume flux sits at $y_7 = 0$ (we are from now on suppressing the indices k and r of the D7- and D3-brane coordinates wherever it does not cause confusion), the attractive force it exerts on a mobile D3-brane only depends on the absolute value, $|y_3|$, of that D3-brane's position [188]. Hence, if we assume that the initial position of the D3-brane has $\text{Im}(y_3) = 0$, the D3-brane is attracted towards the flux D7-brane along the $\text{Re}(y_3)$ direction, which is unaffected by the non-perturbative F-term potential. It should be noted that if the Kähler potential and the relevant gauge couplings had been functions of $|y_3|$ instead of $\text{Im}(y_3)$ (as would be the case, e.g., for a "canonical" Kähler potential $K = |y_3|^2$), one would also have had a shift symmetry along the phase of y_3.[12] However, in that case, also the attractive potential between the D7-brane with world volume flux and the D3-brane would be independent of the phase of y_3, and one would have a completely flat direction and no inflation. It is

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[12]This phase is a compact direction in field space, but so is $\text{Re}(y_3)$ due to the compactness of the torus.

thus important that the shift symmetry is along a direction in field space along which the inflationary potential is not flat.

Now, we will determine this field range for the real part of the canonically normalized D3-brane coordinate, $\phi \equiv \mathrm{Re}(y_3^c)$. Neglecting quantum corrections to the Kähler potential, the kinetic term of $\mathrm{Re}(y_3)$ can be read off from (5.41).

$$M_P^2 \int d^4x \sqrt{\det(\tilde{g}_{\mu\nu})} \tilde{g}^{\mu\nu} \frac{\partial_\mu \mathrm{Re}(y_3) \partial_\nu \mathrm{Re}(y_3)}{4\mathrm{Im}(t)\mathrm{Im}(s) - 2[\mathrm{Im}(y_3)]^2} \ . \tag{5.49}$$

The canonically normalized field, ϕ, is therefore

$$\phi = \frac{M_P \mathrm{Re}(y_3)}{\sqrt{2\mathrm{Im}(t)\mathrm{Im}(s) - [\mathrm{Im}(y_3)]^2}} \ , \tag{5.50}$$

or

$$\phi = M_P \mathrm{Re}(y_3) \sqrt{-\frac{(2\pi)^5 g_s (\alpha')^2}{\mathrm{Vol}_0(K3)\mathrm{Im}(t)}} \ . \tag{5.51}$$

The potential of D-term inflation in the near de Sitter valley where inflationary perturbations are generated is given by a constant term and the Coleman-Weinberg term:

$$V = \frac{g^2\xi^2}{2}\left(1 + \frac{g^2}{16\pi^2}U(x)\right) \ , \tag{5.52}$$

where $x \equiv \frac{\phi}{\sqrt{\xi}}$ and

$$U(x) = (x^2+1)^2\ln(x^2+1) + (x^2-1)^2\ln(x^2-1) - 2x^4\ln(x^2) - 4\ln 2 \ . \tag{5.53}$$

Supersymmetry is broken by the FI parameter ξ, which depends on the 2-form flux on the FI D7-brane. The last term is added to account for the normalization condition $U(1) = 0$, but it can be ignored in our subsequent calculations. Indeed, in the approximation which we are going to use, the corrections to the potential do not affect much its value, $V \approx g^2\xi^2/2$, but these corrections are fully responsible for the value of its derivative V', which does not depend on the last term in (5.53).

Additional quantum corrections: mixture of F- and D-term inflation. As it is well known the gauge coupling constants on the D7-brane will receive non-vanishing quantum corrections, the so-called threshold corrections [203]. They will also affect the non-perturbative superpotential due to gaugino condensation and hence also eventually the inflaton potential. The quantum corrections to the D7-brane gauge coupling break the shift symmetry of the real part of y_3, and in general the real part of y_3 is no longer a distinguished direction. These threshold corrections to gauge coupling constant in orientifold compactifications were computed in [204–210]. For the non-perturbative superpotential only the real part of a holomorphic function is relevant, and for the model under consideration this is given by the modular function ϑ_1 [207, 208]:

$$g^{-2} = \mathrm{Re}(is) - \frac{1}{(2\pi)^2}\mathrm{Re}\,\zeta(y_3, t), \quad \zeta(y_3, t) = \ln\vartheta_1(\sqrt{2\pi}y_3, t) + \ldots, \tag{5.54}$$

where the gauge kinetic function on the D7-branes has the form

$$f_{D7} = is - \frac{1}{8\pi^2}\zeta(y_3 - \mu, t) - \frac{1}{8\pi^2}\zeta(y_3 + \mu, t) + \dots, \tag{5.55}$$

This leads to the following non-perturbative superpotential due to gaugino condensation:

$$W_{\text{n.p.}} = A_0 \exp\left(\frac{8\pi^2 f}{c}\right) = A e^{\frac{8\pi^2}{c}(is - \frac{1}{8\pi^2}\zeta(y_3 - \mu, t) - \frac{1}{8\pi^2}\zeta(y_3 + \mu, t))}, \tag{5.56}$$

where A_0 now might depend on any light charged matter fields and A incorporates in addition an overall factor independent of y_3 coming from the ellipsis in (5.55). Using the explicit form of the string threshold corrections eq. (5.54) we derive

$$W_{\text{n.p.}} = A_0 \exp\left(\frac{8\pi^2 f(M)}{c}\right) = A\left(\vartheta_1\left(\sqrt{2\pi}(y_3 + \mu), t\right)\vartheta_1\left(\sqrt{2\pi}(y_3 - \mu), t\right)\right)^{\frac{-1}{c}} e^{8i\pi^2 s/c}. \tag{5.57}$$

For small values of $y_3 - \mu$ (with $y_3 + \mu$ staying finite) this becomes

$$W_{\text{n.p.}} = A\left(\vartheta_1\left(\sqrt{2\pi}(y_3 + \mu), t\right)\right)^{\frac{-1}{c}} \left((2\pi)^{3/2}\eta(t)^3\right)^{\frac{-1}{c}} (y_3 - \mu)^{\frac{-1}{c}} e^{8i\pi^2 s/c} + \dots. \tag{5.58}$$

This y_3 dependence of the non-perturbative superpotential leads to a non-vanishing F-term potential V_F for the inflaton field ϕ [191]. Neglecting possible quantum corrections to the Kähler potential, and expanding V_F up to quadratic order in ϕ we obtain the following new mass term for the canonically normalized inflaton field:

$$V_F = \frac{|Ae^{-ia\tilde{s}}|^2\tilde{s}_2}{2u_2}\left[\frac{3a^2}{t_2} - 2\phi^2\left(3a\text{Re}(\Delta) + 4t_2|\Delta|^2\right)\right] + \mathcal{O}(\phi^4). \tag{5.59}$$

$$s = \tilde{s} + i\lambda[\text{Re}(y_3)]^2 + \mathcal{O}([\text{Re}(y_3)]^4) \tag{5.60}$$

In order to ensure that the cosmological constant is almost zero after inflation, the first term in (5.59) (a negative contribution to the vacuum energy), has to be canceled. This might require an additional uplifting mechanism.

Here we just assume that the ϕ-independent contribution to the F-term potential is canceled after inflation ends. In that case, the correction due to the F-term potential arising from stringy corrections to the superpotential takes the form

$$V_F = -\frac{m^2}{2}\phi^2, \qquad m^2 = \frac{2|A|^2\tilde{s}_2 e^{2a\tilde{s}}}{u_2}\left[3a\text{Re}(\Delta) + 4t_2|\Delta|^2\right]. \tag{5.61}$$

The function m^2 of (5.61) gets a strong suppression from the exponential pre-factor (note that $\tilde{s}_2$ is negative in our conventions and that $|\tilde{s}_2|$ has to be considerably larger than one in the supergravity regime). Furthermore, also $|u_2|$ is large in the weak coupling limit. In addition, m^2 depends on the complex structure and is thus tunable via a choice of fluxes. Note that even though t_2, u_2 and $\tilde{s}_2$ are all negative in our conventions, m^2 is not necessarily positive, because $\text{Re}(\Delta)$ can have either sign. In figure 17, we plot the function

$$\tilde{m}^2 \equiv 3a\text{Re}(\Delta) + 4t_2|\Delta|^2 \tag{5.62}$$
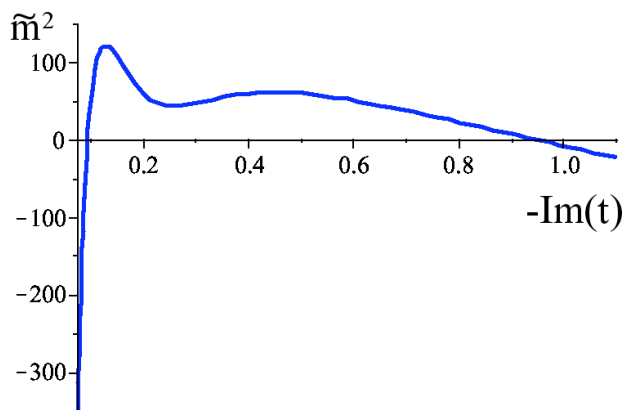
Figure 17. Rescaled mass term for the inflaton fields as a function of the complex structure of the torus T^2.
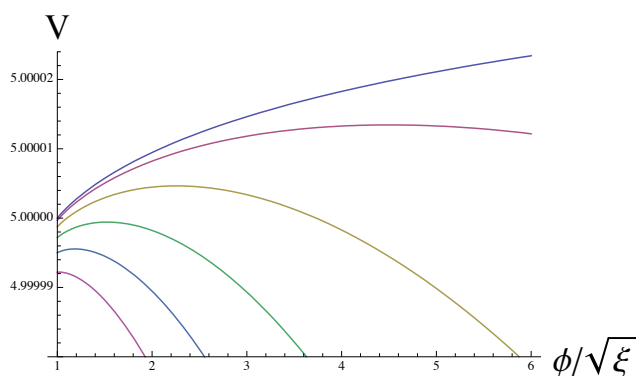
Figure 18. The inflaton potential including the F-term mass corrections. The upper curve is for $m = 0$.

for $a = 8\pi^2/10$ and $\Upsilon = 2\pi^3/30$ as a function of $-t_2$ for the sample value $t_1 = 0.26$. As $\tilde{m}^2 = \gamma m^2$ with $\gamma > 0$, the vanishing of $\tilde{m}^2$ means also a vanishing of m^2. One can show [191] that m^2 can be made small and positive by tuning the parameters t_2 and s_2. Furthermore one can show that quartic corrections to the inflaton can be kept small compared to the mass term considered here.

Thus, the whole D3/D7-brane inflation model potential at small ϕ (i.e. in the regime where inflationary perturbations are generated) in Planck units, and with account of stringy corrections from the stabilizing F-term as explained above, is (see figure 18)

$$V = \frac{g^2 \xi^2}{2}\left(1 + \frac{g^2}{16\pi^2}U\Big(\frac{\phi}{\sqrt{\xi}}\Big)\right) - \frac{m^2}{2}\phi^2\,, \tag{5.63}$$

where $U(x)$ is given in (5.53).

Towards experimental tests of D3/D7 inflation. Let us first discuss the model without quantum corrections, i.e. $m = 0$. As shown in [193] to be in agreement with data

one needs very small coupling constant. Namely for larger couplings $g \geq 2 \times 10^{-3}$ one gets

$$n_s = 1 - 3 \left(\frac{V'}{V}\right)^2 + 2\frac{V''}{V} \approx 1 - \frac{1}{N} \approx 0.98 \ . \tag{5.64}$$

The problem here is that the tension of the cosmic strings produced after inflation in this model is given by

$$G\mu = \frac{\xi}{4} \approx 2.8 \times 10^{-6} \ . \tag{5.65}$$

This is significantly higher than the current bound on the cosmic string tension.

On the other hand for very small couplings, If $g \ll 2 \times 10^{-3}$, we can find a solution

$$n_s = 0.997 \ . \tag{5.66}$$

$$G\mu = 7 \times 10^{-7} \, , \qquad \xi = 2.8 \times 10^{-6} \ . \tag{5.67}$$

That looks very interesting in view of some recent work on cosmic strings and the CMB [151, 152]. According to [152], the recent puzzle of some high l excess power in CMB data from the ACBAR experiment, reported in [211], might possibly be considered as an evidence for the existence of cosmic strings with tensions near the observational bound.

Now consider the case $m^2 \neq 0$. The main result is that these quantum corrections suppress cosmic strings.

Since the analytic solution is known we may try to extract the most important properties of this model concerning the string tension and the spectral index. We have the value of ξ as follows

$$\xi = \frac{2.7 \times 10^{-4} \sqrt{\alpha} \, e^{-\alpha N}}{\pi \sqrt{1 - e^{-\alpha N}}} \, , \tag{5.68}$$

where N can be in the range of 50 to 60. In our estimates we will use, for definiteness, $N = 60$. One now finds the following expression for the spectral index

$$n_s = 1 - \alpha \left(1 + \frac{1}{1 - e^{-\alpha N}}\right) . \tag{5.69}$$

α is related to mass parameter due to quantum corrections:

$$\alpha = \frac{4m^2}{g^2 \xi^2} \ . \tag{5.70}$$

No one can make parametric plots for the values of the string tension versus the spectral index We see that one can suppress the contribution of the cosmic strings by the quantum corrections. In particular, in the limit $\alpha \to 0$, i.e. in the absence of stringy corrections, we get back to $G\mu = \frac{\xi}{4} = 2.8 \times 10^{-6}$ and $n_s = 0.98$ for $N = 60$. In general, a wider range of values for ξ and n_s is possible. Finally note that tensor modes are highly suppressed in this model, since we are dealing with small field inflation.
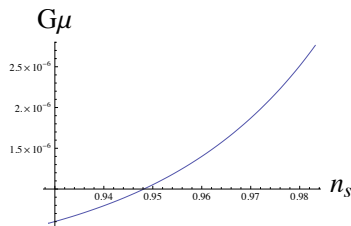
Figure 19. The spectral index n_s versus the cosmic string mass density.

6 Summary

In this review article we discussed several aspects of the string landscape. String theory possesses a huge number of ground states upon compactification to lower dimensions, and hence it is quite evident that the landscape exists. So the question is how to handle it. At the first sight the string landscape leads to an apparent lack of predictive power. Verification as well as falsification of string theory seems almost impossible. Some part of the community in high energy physics takes this against string theory. However the actual situation is not as bad. Intersecting D-branes as well as heterotic string compactifications allow to derive models that come remarkably close to the SM, which is not a priori granted in a fundamental theory that includes gravity and with many constraints like string theory. Of course other low energy worlds with different gauge groups and matter content are also possible. String or D-brane statistics provide some likelihood functions about how often the SM appears in a given ensemble (e.g. closed string background). It is the hope that one one finds statistical correlations telling us that every choice of parameters or gauge groups with associated matter content is possible. The inclusion of background fluxes allows to fix moduli parameters, which enables us to compute couplings in a given flux background. Moreover the smallness of the cosmological constant can be explained by statistical means. Many people object that string statistics is nothing else that the entrance to the anthropic principle. This might be in fact true, and the only explanation for the SM might be the anthropic principle. In other words our universe is only one out of many universes in the cosmic landscape of a so-called multiversum. However, contrary to common believe, the anthropic principle is not completely meaningless in the sense that it still allows for concrete predictions, as predicted by Weinberg [110] for the cosmological constant or recently discussed by Bousso [212] in other instances.

Apart these anthropic considerations, classes of string compactifications still allow for concrete comparisons with experiment. e.g. this is true in large volume compactifications with intermediate string scale of order 10^{11-12} GeV, where one can make definite predictions on the form of the soft SUSY breaking parameters in the MSSM. Even more dramatically, in models where the string scale is around 1 TeV (as an alternative solution of the hierarchy problem to supersymmetry), a large class of D-brane models lead to model independent predictions about the spectrum of string Regge excitations that can be possibly measured at the LHC. These measurements, e.g. in collisions of two gluons into two gluons or into quarks, only see the Regge excitations of the open strings but are insensitive against any details of the internal compact space. Hence in this range of parameters the entire landscape
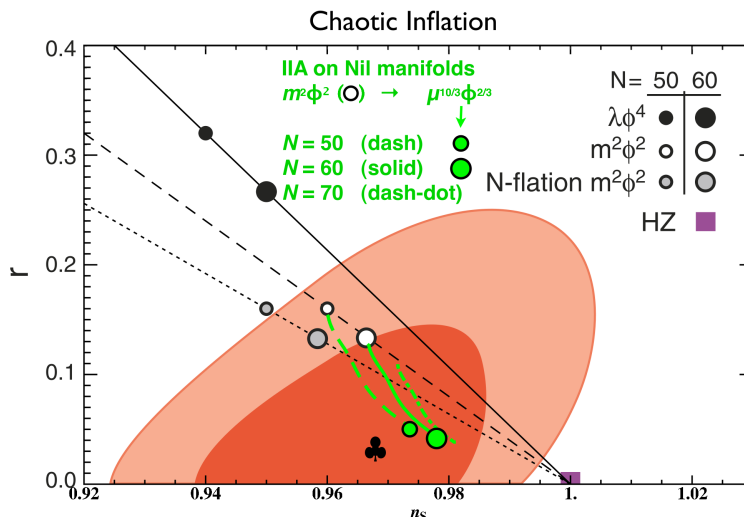
Figure 20. Spectral scalar index versus ratio r in several inflationary scenarios.

is nullified. Other processes like the scattering of two quarks into quarks provide additional informations about the KK spectrum of the underlying geometry. So these processes could provide an image of the internal part of the string landscape. Of course, observing a low string scale at the LHC requires also some big portion of luck, and a low string scale brings many other problems like FCNC's, which have to discussed in a model dependent way. So, we would not be surprises, if the string scale is unfortunately high, after all.

Cosmology provides another promising avenue to make contact between string theory and observations. Here in particular the WMAP5 data about the CMB and future experiments like the PLANCK mission can be used to compare and to constrain string compactifications. This is shown in the following plot, taken from the 5-years report of WMAP [150]. The dots indicate where inflationary models are lying in the n_s, r-plane, like chaotic inflation with quadratic or quartic inflaton potential, or the large inflaton model of type IIA compactification on nilmanifolds. Here, the small field inflation of type IIB on the $K3 \times T^2$ orientifold lies in the low r range, denoted by a ♣ in this plot.

Acknowledgments

References

[1] M.B. Green, J.H. Schwarz and E. Witten, *Superstring theory. Vol. 1 and Vol. 2*, Cambridge Monographs On Mathematical Physics, Cambridge University Press, Cambridge U.K. (1987).

[2] D. Lüst and S. Theisen, *Lectures on string theory*, *Lect. Notes Phys.* **346** (1989) 1 [SPIRES].

[3] J. Polchinski, *String theory. Vol. 1 and Vol. 2*, Cambridge University Press, Cambridge U.K. (1998).

[4] E. Kiritsis, *String theory in a nutshell*, Princeton University Press, Princeton U.S.A. (2007) [SPIRES].

[5] K. Becker, M. Becker and J.H. Schwarz, *String theory and M-theory: A modern introduction*, Cambridge University Press, Cambridge U.K. (2007) [SPIRES].

[6] H. Kawai, D.C. Lewellen and S.H.H. Tye, *Construction of Four-Dimensional Fermionic String Models*, *Phys. Rev. Lett.* **57** (1986) 1832 [*Erratum ibid.* **58** (1987) 429] [SPIRES].

[7] W. Lerche, D. Lüst and A.N. Schellekens, *Chiral Four-Dimensional Heterotic Strings from Selfdual Lattices*, *Nucl. Phys.* **B 287** (1987) 477 [SPIRES].

[8] I. Antoniadis, C.P. Bachas and C. Kounnas, *Four-Dimensional Superstrings*, *Nucl. Phys.* **B 289** (1987) 87 [SPIRES].

[9] R. Bousso and J. Polchinski, *Quantization of four-form fluxes and dynamical neutralization of the cosmological constant*, *JHEP* **06** (2000) 006 [hep-th/0004134] [SPIRES].

[10] M.R. Douglas, *The statistics of string/M theory vacua*, *JHEP* **05** (2003) 046 [hep-th/0303194] [SPIRES].

[11] L. Susskind, *The anthropic landscape of string theory*, hep-th/0302219 [SPIRES].

[12] A.N. Schellekens, *The Emperor's Last Clothes?*, *Rept. Prog. Phys.* **71** (2008) 072201 [arXiv:0807.3249] [SPIRES].

[13] A. Linde, *Inflationary Cosmology*, *Lect. Notes Phys.* **738** (2008) 1 [arXiv:0705.0164] [SPIRES].

[14] B. de Carlos, J.A. Casas, F. Quevedo and E. Roulet, *Model independent properties and cosmological implications of the dilaton and moduli sectors of $4 - D$ strings*, *Phys. Lett.* **B 318** (1993) 447 [hep-ph/9308325] [SPIRES].

[15] C. Bachas, *A Way to break supersymmetry*, hep-th/9503030 [SPIRES].

[16] R. Blumenhagen, L. Görlich, B. Körs and D. Lüst, *Noncommutative compactifications of type-I strings on tori with magnetic background flux*, *JHEP* **10** (2000) 006 [hep-th/0007024] [SPIRES].

[17] C. Angelantonj, I. Antoniadis, E. Dudas and A. Sagnotti, *Type-I strings on magnetised orbifolds and brane transmutation*, *Phys. Lett.* **B 489** (2000) 223 [hep-th/0007090] [SPIRES].

[18] G. Aldazabal, S. Franco, L.E. Ibáñez, R. Rabadán and A.M. Uranga, $D = 4$ *chiral string compactifications from intersecting branes*, *J. Math. Phys.* **42** (2001) 3103 [hep-th/0011073] [SPIRES].

[19] M. Cvetič, G. Shiu and A.M. Uranga, *Three-family supersymmetric standard like models from intersecting brane worlds*, *Phys. Rev. Lett.* **87** (2001) 201801 [hep-th/0107143] [SPIRES].

[20] E. Kiritsis, *D-branes in standard model building, gravity and cosmology*, *Fortsch. Phys.* **52** (2004) 200 [*Phys. Rept.* **421** (2005) 105] [*Erratum ibid.* **429** (2006) 121] [`hep-th/0310001`] [SPIRES].

[21] D. Lüst, *Intersecting brane worlds: a path to the standard model?*, *Class. Quant. Grav.* **21** (2004) S1399 [`hep-th/0401156`] [SPIRES].

[22] R. Blumenhagen, M. Cvetič, P. Langacker and G. Shiu, *Toward realistic intersecting D-brane models*, *Ann. Rev. Nucl. Part. Sci.* **55** (2005) 71 [`hep-th/0502005`] [SPIRES].

[23] R. Blumenhagen, B. Körs, D. Lüst and S. Stieberger, *Four-dimensional String Compactifications with D-branes, Orientifolds and Fluxes*, *Phys. Rept.* **445** (2007) 1 [`hep-th/0610327`] [SPIRES].

[24] F. Marchesano, *Progress in D-brane model building*, *Fortsch. Phys.* **55** (2007) 491 [`hep-th/0702094`] [SPIRES].

[25] R. Blumenhagen, V. Braun, T.W. Grimm and T. Weigand, *GUTs in Type IIB Orientifold Compactifications*, `arXiv:0811.2936` [SPIRES].

[26] C. Beasley, J.J. Heckman and C. Vafa, *GUTs and Exceptional Branes in F-theory. I*, *JHEP* **01** (2009) 058 [`arXiv:0802.3391`] [SPIRES].

[27] C. Beasley, J.J. Heckman and C. Vafa, *GUTs and Exceptional Branes in F-theory. II: experimental Predictions*, *JHEP* **01** (2009) 059 [`arXiv:0806.0102`] [SPIRES].

[28] D. Berenstein, V. Jejjala and R.G. Leigh, *The standard model on a D-brane*, *Phys. Rev. Lett.* **88** (2002) 071602 [`hep-ph/0105042`] [SPIRES].

[29] H. Verlinde and M. Wijnholt, *Building the Standard Model on a D3-brane*, *JHEP* **01** (2007) 106 [`hep-th/0508089`] [SPIRES].

[30] D. Malyshev and H. Verlinde, *D-branes at Singularities and String Phenomenology*, *Nucl. Phys. Proc. Suppl.* **171** (2007) 139 [`arXiv:0711.2451`] [SPIRES].

[31] I. Antoniadis, E. Kiritsis and T.N. Tomaras, *A D-brane alternative to unification*, *Phys. Lett.* **B 486** (2000) 186 [`hep-ph/0004214`] [SPIRES].

[32] R. Blumenhagen, F. Gmeiner, G. Honecker, D. Lüst and T. Weigand, *The statistics of supersymmetric D-brane models*, *Nucl. Phys.* **B 713** (2005) 83 [`hep-th/0411173`] [SPIRES].

[33] F. Gmeiner, R. Blumenhagen, G. Honecker, D. Lüst and T. Weigand, *One in a billion: MSSM-like D-brane statistics*, *JHEP* **01** (2006) 004 [`hep-th/0510170`] [SPIRES].

[34] F. Gmeiner, *Standard model statistics of a type-II orientifold*, *Fortsch. Phys.* **54** (2006) 391 [`hep-th/0512190`] [SPIRES].

[35] F. Gmeiner, *Gauge sector statistics of intersecting D-brane models*, *Fortsch. Phys.* **55** (2007) 111 [`hep-th/0608227`] [SPIRES].

[36] M.R. Douglas and W. Taylor, *The landscape of intersecting brane models*, *JHEP* **01** (2007) 031 [`hep-th/0606109`] [SPIRES].

[37] F. Marchesano and G. Shiu, *Building MSSM flux vacua*, *JHEP* **11** (2004) 041 [`hep-th/0409132`] [SPIRES].

[38] F. Gmeiner, D. Lüst and M. Stein, *Statistics of intersecting D-brane models on T^6/Z_6*, *JHEP* **05** (2007) 018 [`hep-th/0703011`] [SPIRES].

[39] F. Gmeiner and G. Honecker, *Mapping an Island in the Landscape*, *JHEP* **09** (2007) 128 [`arXiv:0708.2285`] [SPIRES].

[40] F. Gmeiner and G. Honecker, *Millions of Standard Models on Z6-prime?*, *JHEP* **07** (2008) 052 [`arXiv:0806.3039`] [SPIRES].

[41] F. Gmeiner, *Statistical analysis of a subset of the string theory landscape*, `arXiv:0810.3623` [SPIRES].

[42] G. Honecker and T. Ott, *Getting just the supersymmetric standard model at intersecting branes on the Z(6)-orientifold*, *Phys. Rev.* **D 70** (2004) 126010 [*Erratum ibid.* **D 71** (2005) 069902] [`hep-th/0404055`] [SPIRES].

[43] N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *The hierarchy problem and new dimensions at a millimeter*, *Phys. Lett.* **B 429** (1998) 263 [`hep-ph/9803315`] [SPIRES].

[44] I. Antoniadis, N. Arkani-Hamed, S. Dimopoulos and G.R. Dvali, *New dimensions at a millimeter to aFermi and superstrings at a TeV*, *Phys. Lett.* **B 436** (1998) 257 [`hep-ph/9804398`] [SPIRES].

[45] V. Balasubramanian, P. Berglund, J.P. Conlon and F. Quevedo, *Systematics of Moduli Stabilisation in Calabi-Yau Flux Compactifications*, *JHEP* **03** (2005) 007 [`hep-th/0502058`] [SPIRES].

[46] J.P. Conlon, F. Quevedo and K. Suruliz, *Large-volume flux compactifications: moduli spectrum and D3/D7 soft supersymmetry breaking*, *JHEP* **08** (2005) 007 [`hep-th/0505076`] [SPIRES].

[47] J.P. Conlon, C.H. Kom, K. Suruliz, B.C. Allanach and F. Quevedo, *Sparticle Spectra and LHC Signatures for Large Volume String Compactifications*, *JHEP* **08** (2007) 061 [`arXiv:0704.3403`] [SPIRES].

[48] R. Blumenhagen, M. Cvetič, F. Marchesano and G. Shiu, *Chiral D-brane models with frozen open string moduli*, *JHEP* **03** (2005) 050 [`hep-th/0502095`] [SPIRES].

[49] S.B. Giddings and S.D. Thomas, *High energy colliders as black hole factories: the end of short distance physics*, *Phys. Rev.* **D 65** (2002) 056010 [`hep-ph/0106219`] [SPIRES].

[50] S. Dimopoulos and G.L. Landsberg, *Black Holes at the LHC*, *Phys. Rev. Lett.* **87** (2001) 161602 [`hep-ph/0106295`] [SPIRES].

[51] P. Meade and L. Randall, *Black Holes and Quantum Gravity at the LHC*, *JHEP* **05** (2008) 003 [`arXiv:0708.3017`] [SPIRES].

[52] E. Kiritsis and P. Anastasopoulos, *The anomalous magnetic moment of the muon in the D-brane realization of the standard model*, *JHEP* **05** (2002) 054 [`hep-ph/0201295`] [SPIRES].

[53] D.M. Ghilencea, L.E. Ibáñez, N. Irges and F. Quevedo, *TeV-Scale Z' Bosons from D-branes*, *JHEP* **08** (2002) 016 [`hep-ph/0205083`] [SPIRES].

[54] S.A. Abel, M.D. Goodsell, J. Jaeckel, V.V. Khoze and A. Ringwald, *Kinetic Mixing of the Photon with Hidden U(1)s in String Phenomenology*, *JHEP* **07** (2008) 124 [`arXiv:0803.1449`] [SPIRES].

[55] S. Cullen, M. Perelstein and M.E. Peskin, *TeV strings and collider probes of large extra dimensions*, *Phys. Rev.* **D 62** (2000) 055012 [`hep-ph/0001166`] [SPIRES].

[56] L.A. Anchordoqui, H. Goldberg, S. Nawata and T.R. Taylor, *Direct photons as probes of low mass strings at the LHC*, *Phys. Rev.* **D 78** (2008) 016005 [`arXiv:0804.2013`] [SPIRES].

[57] D. Lüst, S. Stieberger and T.R. Taylor, *The LHC String Hunter's Companion*, *Nucl. Phys.* **B 808** (2009) 1 [`arXiv:0807.3333`] [SPIRES].

[58] L.A. Anchordoqui et al., *Dijet signals for low mass strings at the LHC*, *Phys. Rev. Lett.* **101** (2008) 241803 [arXiv:0808.0497] [SPIRES].

[59] S. Stieberger and T.R. Taylor, *Amplitude for N-gluon superstring scattering*, *Phys. Rev. Lett.* **97** (2006) 211601 [hep-th/0607184] [SPIRES].

[60] S. Stieberger and T.R. Taylor, *Multi-gluon scattering in open superstring theory*, *Phys. Rev.* **D 74** (2006) 126007 [hep-th/0609175] [SPIRES].

[61] M. Cvetič and I. Papadimitriou, *Conformal field theory couplings for intersecting D-branes on orientifolds*, *Phys. Rev.* **D 68** (2003) 046001 [*Erratum ibid.* **D 70** (2004) 029903] [hep-th/0303083] [SPIRES].

[62] I.R. Klebanov and E. Witten, *Proton decay in intersecting D-brane models*, *Nucl. Phys.* **B 664** (2003) 3 [hep-th/0304079] [SPIRES].

[63] S.A. Abel and A.W. Owen, *N-point amplitudes in intersecting brane models*, *Nucl. Phys.* **B 682** (2004) 183 [hep-th/0310257] [SPIRES].

[64] S.A. Abel, O. Lebedev and J. Santiago, *Flavour in intersecting brane models and bounds on the string scale*, *Nucl. Phys.* **B 696** (2004) 141 [hep-ph/0312157] [SPIRES].

[65] M. Cvetič and R. Richter, *Proton decay via dimension-six operators in intersecting D6-brane models*, *Nucl. Phys.* **B 762** (2007) 112 [hep-th/0606001] [SPIRES].

[66] L.A. Anchordoqui, H. Goldberg and T.R. Taylor, *Decay widths of lowest massive Regge excitations of open strings*, *Phys. Lett.* **B 668** (2008) 373 [arXiv:0806.3420] [SPIRES].

[67] J. Pumplin et al., *New generation of parton distributions with uncertainties from global QCD analysis*, *JHEP* **07** (2002) 012 [hep-ph/0201195] [SPIRES].

[68] A. Bhatti et al., *CMS search plans and sensitivity to new physics with dijets*, *J. Phys.* **G 36** (2009) 015004 [arXiv:0807.4961] [SPIRES].

[69] M. Graña, *Flux compactifications in string theory: a comprehensive review*, *Phys. Rept.* **423** (2006) 91 [hep-th/0509003] [SPIRES].

[70] F. Denef, M.R. Douglas and S. Kachru, *Physics of string flux compactifications*, *Ann. Rev. Nucl. Part. Sci.* **57** (2007) 119 [hep-th/0701050] [SPIRES].

[71] S. Ashok and M.R. Douglas, *Counting flux vacua*, *JHEP* **01** (2004) 060 [hep-th/0307049] [SPIRES].

[72] F. Denef and M.R. Douglas, *Distributions of flux vacua*, *JHEP* **05** (2004) 072 [hep-th/0404116] [SPIRES].

[73] M. Dine, E. Gorbatov and S.D. Thomas, *Low energy supersymmetry from the landscape*, *JHEP* **08** (2008) 098 [hep-th/0407043] [SPIRES].

[74] N. Arkani-Hamed and S. Dimopoulos, *Supersymmetric unification without low energy supersymmetry and signatures for fine-tuning at the LHC*, *JHEP* **06** (2005) 073 [hep-th/0405159] [SPIRES].

[75] F. Denef and M.R. Douglas, *Distributions of nonsupersymmetric flux vacua*, *JHEP* **03** (2005) 061 [hep-th/0411183] [SPIRES].

[76] N. Arkani-Hamed, S. Dimopoulos and S. Kachru, *Predictive landscapes and new physics at a TeV*, hep-th/0501082 [SPIRES].

[77] D. Andriot, *New supersymmetric flux vacua with intermediate SU(2) structure*, *JHEP* **08** (2008) 096 [arXiv:0804.1769] [SPIRES].

[78] C. Caviezel et al., *The effective theory of type IIA AdS4 compactifications on nilmanifolds and cosets*, *Class. Quant. Grav.* **26** (2009) 025014 [`arXiv:0806.3458`] [SPIRES].

[79] S. Chiossi and S. Salamon, *The intrinsic torsion of $SU(3)$ and G_2 structures*, `math.DG/0202282` [SPIRES].

[80] S. Gurrieri, J. Louis, A. Micu and D. Waldram, *Mirror symmetry in generalized Calabi-Yau compactifications*, *Nucl. Phys.* **B 654** (2003) 61 [`hep-th/0211102`] [SPIRES].

[81] G. Lopes Cardoso et al., *Non-Kähler string backgrounds and their five torsion classes*, *Nucl. Phys.* **B 652** (2003) 5 [`hep-th/0211118`] [SPIRES].

[82] M. Graña, R. Minasian, M. Petrini and A. Tomasiello, *Supersymmetric backgrounds from generalized Calabi-Yau manifolds*, *JHEP* **08** (2004) 046 [`hep-th/0406137`] [SPIRES].

[83] M. Graña, R. Minasian, A. Tomasiello and M. Petrini, *Supersymmetric backgrounds from generalized Calabi-Yau manifolds*, *Fortsch. Phys.* **53** (2005) 885 [SPIRES].

[84] T.H. Buscher, *A Symmetry of the String Background Field Equations*, *Phys. Lett.* **B 194** (1987) 59 [SPIRES].

[85] S. Kachru, M.B. Schulz, P.K. Tripathy and S.P. Trivedi, *New supersymmetric string compactifications*, *JHEP* **03** (2003) 061 [`hep-th/0211182`] [SPIRES].

[86] C.M. Hull and R.A. Reid-Edwards, *Flux compactifications of string theory on twisted tori*, `hep-th/0503114` [SPIRES].

[87] M. Graña, R. Minasian, M. Petrini and A. Tomasiello, *A scan for new $N = 1$ vacua on twisted tori*, *JHEP* **05** (2007) 031 [`hep-th/0609124`] [SPIRES].

[88] J. Shelton, W. Taylor and B. Wecht, *Nongeometric Flux Compactifications*, *JHEP* **10** (2005) 085 [`hep-th/0508133`] [SPIRES].

[89] A. Dabholkar and C. Hull, *Generalised T-duality and non-geometric backgrounds*, *JHEP* **05** (2006) 009 [`hep-th/0512005`] [SPIRES].

[90] J. Shelton, W. Taylor and B. Wecht, *Generalized flux vacua*, *JHEP* **02** (2007) 095 [`hep-th/0607015`] [SPIRES].

[91] B. Wecht, *Lectures on Nongeometric Flux Compactifications*, *Class. Quant. Grav.* **24** (2007) S773 [`arXiv:0708.3984`] [SPIRES].

[92] C.M. Hull, *Doubled geometry and T-folds*, *JHEP* **07** (2007) 080 [`hep-th/0605149`] [SPIRES].

[93] M. Graña, *Flux compactifications and generalized geometries*, *Class. Quant. Grav.* **23** (2006) S883 [SPIRES].

[94] M. Graña, J. Louis and D. Waldram, *$SU(3) \times SU(3)$ compactification and mirror duals of magnetic fluxes*, *JHEP* **04** (2007) 101 [`hep-th/0612237`] [SPIRES].

[95] M. Graña, R. Minasian, M. Petrini and D. Waldram, *T-duality, Generalized Geometry and Non-Geometric Backgrounds*, `arXiv:0807.4527` [SPIRES].

[96] T.W. Grimm and J. Louis, *The effective action of $N = 1$ Calabi-Yau orientifolds*, *Nucl. Phys.* **B 699** (2004) 387 [`hep-th/0403067`] [SPIRES].

[97] M. Graña, J. Louis and D. Waldram, *Hitchin functionals in $N = 2$ supergravity*, *JHEP* **01** (2006) 008 [`hep-th/0505264`] [SPIRES].

[98] I. Benmachiche and T.W. Grimm, *Generalized $N = 1$ orientifold compactifications and the Hitchin functionals*, *Nucl. Phys.* **B 748** (2006) 200 [`hep-th/0602241`] [SPIRES].

[99] C. Jeschek and F. Witt, *Generalised geometries, constrained critical points and Ramond-Ramond fields*, math.DG/0510131 [SPIRES].

[100] F. Gmeiner and F. Witt, *Calibrations and T-duality*, *Commun. Math. Phys.* **283** (2008) 543 [math.DG/0605710] [SPIRES].

[101] F. Gmeiner and F. Witt, *Calibrations on spaces with G x G structure*, *Fortsch. Phys.* **55** (2007) 727 [hep-th/0701109] [SPIRES].

[102] D. Cassani and A. Bilal, *Effective actions and $N = 1$ vacuum conditions from $SU(3) \times SU(3)$ compactifications*, *JHEP* **09** (2007) 076 [arXiv:0707.3125] [SPIRES].

[103] P. Koerber and L. Martucci, *From ten to four and back again: how to generalize the geometry*, *JHEP* **08** (2007) 059 [arXiv:0707.1038] [SPIRES].

[104] S. Gukov, C. Vafa and E. Witten, *CFT's from Calabi-Yau four-folds*, *Nucl. Phys.* **B 584** (2000) 69 [*Erratum ibid.* **B 608** (2001) 477] [hep-th/9906070] [SPIRES].

[105] T.R. Taylor and C. Vafa, *RR flux on Calabi-Yau and partial supersymmetry breaking*, *Phys. Lett.* **B 474** (2000) 130 [hep-th/9912152] [SPIRES].

[106] D. Cassani, *Reducing democratic type-II supergravity on $SU(3) \times SU(3)$ structures*, *JHEP* **06** (2008) 027 [arXiv:0804.0595] [SPIRES].

[107] P. Mayr, *On supersymmetry breaking in string theory and its realization in brane worlds*, *Nucl. Phys.* **B 593** (2001) 99 [hep-th/0003198] [SPIRES].

[108] G. Curio, A. Klemm, D. Lüst and S. Theisen, *On the vacuum structure of type-II string compactifications on Calabi-Yau spaces with H-fluxes*, *Nucl. Phys.* **B 609** (2001) 3 [hep-th/0012213] [SPIRES].

[109] S.B. Giddings, S. Kachru and J. Polchinski, *Hierarchies from fluxes in string compactifications*, *Phys. Rev.* **D 66** (2002) 106006 [hep-th/0105097] [SPIRES].

[110] S. Weinberg, *Anthropic Bound on the Cosmological Constant*, *Phys. Rev. Lett.* **59** (1987) 2607 [SPIRES].

[111] S. Kachru, R. Kallosh, A. Linde and S.P. Trivedi, *de Sitter vacua in string theory*, *Phys. Rev.* **D 68** (2003) 046005 [hep-th/0301240] [SPIRES].

[112] P.G. Camara, L.E. Ibáñez and A.M. Uranga, *Flux-induced SUSY-breaking soft terms*, *Nucl. Phys.* **B 689** (2004) 195 [hep-th/0311241] [SPIRES].

[113] M. Graña, T.W. Grimm, H. Jockers and J. Louis, *Soft Supersymmetry Breaking in Calabi-Yau Orientifolds with D-branes and Fluxes*, *Nucl. Phys.* **B 690** (2004) 21 [hep-th/0312232] [SPIRES].

[114] D. Lüst, S. Reffert and S. Stieberger, *Flux-induced Soft Supersymmetry Breaking in Chiral Type IIB Orientifolds with D3/D7-branes*, *Nucl. Phys.* **B 706** (2005) 3 [hep-th/0406092] [SPIRES].

[115] P.G. Camara, L.E. Ibáñez and A.M. Uranga, *Flux-induced SUSY-breaking soft terms on D7-D3 brane systems*, *Nucl. Phys.* **B 708** (2005) 268 [hep-th/0408036] [SPIRES].

[116] D. Lüst, S. Reffert and S. Stieberger, *MSSM with soft SUSY breaking terms from D7-branes with fluxes*, *Nucl. Phys.* **B 727** (2005) 264 [hep-th/0410074] [SPIRES].

[117] K. Choi, A. Falkowski, H.P. Nilles and M. Olechowski, *Soft supersymmetry breaking in KKLT flux compactification*, *Nucl. Phys.* **B 718** (2005) 113 [hep-th/0503216] [SPIRES].

[118] J.P. Conlon and F. Quevedo, *Gaugino and scalar masses in the landscape*, *JHEP* **06** (2006) 029 [hep-th/0605141] [SPIRES].

[119] J.P. Conlon, S.S. AbdusSalam, F. Quevedo and K. Suruliz, *Soft SUSY breaking terms for chiral matter in IIB string compactifications*, *JHEP* **01** (2007) 032 [hep-th/0610129] [SPIRES].

[120] E. Witten, *Non-Perturbative Superpotentials In String Theory*, *Nucl. Phys.* **B 474** (1996) 343 [hep-th/9604030] [SPIRES].

[121] T.R. Taylor, G. Veneziano and S. Yankielowicz, *Supersymmetric QCD and Its Massless Limit: an Effective Lagrangian Analysis*, *Nucl. Phys.* **B 218** (1983) 493 [SPIRES].

[122] I. Affleck, M. Dine and N. Seiberg, *Supersymmetry Breaking by Instantons*, *Phys. Rev. Lett.* **51** (1983) 1026 [SPIRES].

[123] D. Lüst, S. Reffert, W. Schulgin and S. Stieberger, *Moduli stabilization in type IIB orientifolds. I: orbifold limits*, *Nucl. Phys.* **B 766** (2007) 68 [hep-th/0506090] [SPIRES].

[124] F. Denef, M.R. Douglas, B. Florea, A. Grassi and S. Kachru, *Fixing all moduli in a simple F-theory compactification*, *Adv. Theor. Math. Phys.* **9** (2005) 861 [hep-th/0503124] [SPIRES].

[125] D. Lüst, S. Reffert, E. Scheidegger, W. Schulgin and S. Stieberger, *Moduli stabilization in type IIB orientifolds. II*, *Nucl. Phys.* **B 766** (2007) 178 [hep-th/0609013] [SPIRES].

[126] D. Lüst, S. Reffert, E. Scheidegger and S. Stieberger, *Resolved toroidal orbifolds and their orientifolds*, *Adv. Theor. Math. Phys.* **12** (2008) 67 [hep-th/0609014] [SPIRES].

[127] D. Lüst, P. Mayr, R. Richter and S. Stieberger, *Scattering of gauge, matter and moduli fields from intersecting branes*, *Nucl. Phys.* **B 696** (2004) 205 [hep-th/0404134] [SPIRES].

[128] D. Lüst, P. Mayr, S. Reffert and S. Stieberger, *F-theory flux, destabilization of orientifolds and soft terms on D7-branes*, *Nucl. Phys.* **B 732** (2006) 243 [hep-th/0501139] [SPIRES].

[129] R. Blumenhagen, S. Moster and E. Plauschinn, *Moduli Stabilisation versus Chirality for MSSM like Type IIB Orientifolds*, *JHEP* **01** (2008) 058 [arXiv:0711.3389] [SPIRES].

[130] K. Behrndt and M. Cvetič, *Anti-de Sitter vacua of gauged supergravities with 8 supercharges*, *Phys. Rev.* **D 61** (2000) 101901 [hep-th/0001159] [SPIRES].

[131] K. Behrndt and M. Cvetič, *General $N = 1$ Supersymmetric Flux Vacua of (Massive) Type IIA String Theory*, *Phys. Rev. Lett.* **95** (2005) 021601 [hep-th/0403049] [SPIRES].

[132] J.-P. Derendinger, C. Kounnas, P.M. Petropoulos and F. Zwirner, *Superpotentials in IIA compactifications with general fluxes*, *Nucl. Phys.* **B 715** (2005) 211 [hep-th/0411276] [SPIRES].

[133] D. Lüst and D. Tsimpis, *Supersymmetric AdS_4 compactifications of IIA supergravity*, *JHEP* **02** (2005) 027 [hep-th/0412250] [SPIRES].

[134] G. Villadoro and F. Zwirner, *$N = 1$ effective potential from dual type-IIA D6/O6 orientifolds with general fluxes*, *JHEP* **06** (2005) 047 [hep-th/0503169] [SPIRES].

[135] O. DeWolfe, A. Giryavets, S. Kachru and W. Taylor, *Type IIA moduli stabilization*, *JHEP* **07** (2005) 066 [hep-th/0505160] [SPIRES].

[136] P.G. Camara, A. Font and L.E. Ibáñez, *Fluxes, moduli fixing and MSSM-like vacua in a simple IIA orientifold*, *JHEP* **09** (2005) 013 [hep-th/0506066] [SPIRES].

[137] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, *$N = 6$ superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, *JHEP* **10** (2008) 091 [arXiv:0806.1218] [SPIRES].

[138] L.J. Romans, *Massive N=2a Supergravity in Ten-Dimensions*, *Phys. Lett.* **B 169** (1986) 374 [SPIRES].

[139] B.S. Acharya, F. Benini and R. Valandro, *Fixing moduli in exact type IIA flux vacua*, *JHEP* **02** (2007) 018 [hep-th/0607223] [SPIRES].

[140] G. Villadoro and F. Zwirner, *On general flux backgrounds with localized sources*, *JHEP* **11** (2007) 082 [arXiv:0710.2551] [SPIRES].

[141] G. Aldazabal and A. Font, *A second look at $N = 1$ supersymmetric AdS_4 vacua of type IIA supergravity*, *JHEP* **02** (2008) 086 [arXiv:0712.1021] [SPIRES].

[142] A. Tomasiello, *New string vacua from twistor spaces*, *Phys. Rev.* **D 78** (2008) 046007 [arXiv:0712.1396] [SPIRES].

[143] P. Koerber, D. Lüst and D. Tsimpis, *Type IIA AdS4 compactifications on cosets, interpolations and domain walls*, *JHEP* **07** (2008) 017 [arXiv:0804.0614] [SPIRES].

[144] G. Villadoro and F. Zwirner, *Beyond Twisted Tori*, *Phys. Lett.* **B 652** (2007) 118 [arXiv:0706.3049] [SPIRES].

[145] C. Kounnas, D. Lüst, P.M. Petropoulos and D. Tsimpis, *AdS4 flux vacua in type-II superstrings and their domain- wall solutions*, *JHEP* **09** (2007) 051 [arXiv:0707.4270] [SPIRES].

[146] M. Cvetič, F. Quevedo and S.-J. Rey, *Stringy domain walls and target space modular invariance*, *Phys. Rev. Lett.* **67** (1991) 1836 [SPIRES].

[147] M. Cvetič and H.H. Soleng, *Supergravity domain walls*, *Phys. Rept.* **282** (1997) 159 [hep-th/9604090] [SPIRES].

[148] A. Ceresole, G. Dall'Agata, A. Giryavets, R. Kallosh and A. Linde, *Domain walls, near-BPS bubbles and probabilities in the landscape*, *Phys. Rev.* **D 74** (2006) 086010 [hep-th/0605266] [SPIRES].

[149] S.R. Coleman and F. De Luccia, *Gravitational Effects on and of Vacuum Decay*, *Phys. Rev.* **D 21** (1980) 3305 [SPIRES].

[150] WMAP collaboration, E. Komatsu et al., *Five-Year Wilkinson Microwave Anisotropy Probe (WMAP 1) Observations: Cosmological Interpretation*, *Astrophys. J. Suppl.* **180** (2009) 330 [arXiv:0803.0547] [SPIRES].

[151] N. Bevis, M. Hindmarsh, M. Kunz and J. Urrestilla, *Fitting CMB data with cosmic strings and inflation*, *Phys. Rev. Lett.* **100** (2008) 021301 [astro-ph/0702223] [SPIRES].

[152] L. Pogosian, S.H.H. Tye, I. Wasserman and M. Wyman, *Cosmic Strings as the Source of Small-Scale Microwave Background Anisotropy*, *JCAP* **02** (2009) 013 [arXiv:0804.0810] [SPIRES].

[153] V. Mukhanov, *Physical foundations of cosmology*, Cambridge University Press, Cambridge U.K. (2005) [SPIRES].

[154] A.D. Linde, *Chaotic Inflation*, *Phys. Lett.* **B 129** (1983) 177 [SPIRES].

[155] D.H. Lyth, *What would we learn by detecting a gravitational wave signal in the cosmic microwave background anisotropy?*, *Phys. Rev. Lett.* **78** (1997) 1861 [hep-ph/9606387] [SPIRES].

[156] P. Binetruy and G.R. Dvali, *D-term inflation*, *Phys. Lett.* **B 388** (1996) 241 [hep-ph/9606342] [SPIRES].

[157] E. Halyo, *Hybrid inflation from supergravity D-terms*, *Phys. Lett.* **B 387** (1996) 43 [`hep-ph/9606423`] [SPIRES].

[158] A.D. Linde, *Hybrid inflation*, *Phys. Rev.* **D 49** (1994) 748 [`astro-ph/9307002`] [SPIRES].

[159] G. Dvali, *Black Holes and Large-N Species Solution to the Hierarchy Problem*, `arXiv:0706.2050` [SPIRES].

[160] C. Vafa, *The string landscape and the swampland*, `hep-th/0509212` [SPIRES].

[161] N. Arkani-Hamed, L. Motl, A. Nicolis and C. Vafa, *The string landscape, black holes and gravity as the weakest force*, *JHEP* **06** (2007) 060 [`hep-th/0601001`] [SPIRES].

[162] G. Dvali and D. Lüst, *Power of Black Hole Physics: seeing through the Vacuum Landscape*, `arXiv:0801.1287` [SPIRES].

[163] E.J. Copeland, A.R. Liddle, D.H. Lyth, E.D. Stewart and D. Wands, *False vacuum inflation with Einstein gravity*, *Phys. Rev.* **D 49** (1994) 6410 [`astro-ph/9401011`] [SPIRES].

[164] G.R. Dvali, Q. Shafi and R.K. Schaefer, *Large scale structure and supersymmetric inflation without fine tuning*, *Phys. Rev. Lett.* **73** (1994) 1886 [`hep-ph/9406319`] [SPIRES].

[165] S.H. Henry Tye, *Brane inflation: string theory viewed from the cosmos*, Lect. Notes Phys. **737** (2008) 949 [`hep-th/0610221`] [SPIRES].

[166] J.M. Cline, *String cosmology*, `hep-th/0612129` [SPIRES].

[167] R. Kallosh, *On Inflation in String Theory*, *Lect. Notes Phys.* **738** (2008) 119 [`hep-th/0702059`] [SPIRES].

[168] C.P. Burgess, *Lectures on Cosmic Inflation and its Potential Stringy Realizations*, *PoS* **P 2GC** (2006) 008 [`arXiv:0708.2865`] [SPIRES].

[169] L. McAllister and E. Silverstein, *String Cosmology: a Review*, *Gen. Rel. Grav.* **40** (2008) 565 [`arXiv:0710.2951`] [SPIRES].

[170] G.R. Dvali and S.H.H. Tye, *Brane inflation*, *Phys. Lett.* **B 450** (1999) 72 [`hep-ph/9812483`] [SPIRES].

[171] C.P. Burgess et al., *The Inflationary Brane-Antibrane Universe*, *JHEP* **07** (2001) 047 [`hep-th/0105204`] [SPIRES].

[172] G. Dvali, R. Kallosh and A. Van Proeyen, *D-term strings*, *JHEP* **01** (2004) 035 [`hep-th/0312005`] [SPIRES].

[173] C. Caviezel, P. Koeber, S. Körs, D. Lüst, T. Wrase and M. Zagermann, *On the Cosmology of Type IIA Compactifications on SU(3) Structure Manifolds*, to appear.

[174] M.P. Hertzberg, M. Tegmark, S. Kachru, J. Shelton and O. Ozcan, *Searching for Inflation in Simple String Theory Models: an Astrophysical Perspective*, *Phys. Rev.* **D 76** (2007) 103521 [`arXiv:0709.0002`] [SPIRES].

[175] M.P. Hertzberg, S. Kachru, W. Taylor and M. Tegmark, *Inflationary Constraints on Type IIA String Theory*, *JHEP* **12** (2007) 095 [`arXiv:0711.2512`] [SPIRES].

[176] E. Silverstein, *Simple de Sitter Solutions*, *Phys. Rev.* **D 77** (2008) 106006 [`arXiv:0712.1196`] [SPIRES].

[177] E. Silverstein and A. Westphal, *Monodromy in the CMB: gravity Waves and String Inflation*, *Phys. Rev.* **D 78** (2008) 106003 [`arXiv:0803.3085`] [SPIRES].

[178] L. McAllister, E. Silverstein and A. Westphal, *Gravity Waves and Linear Inflation from Axion Monodromy*, `arXiv:0808.0706` [SPIRES].

[179] S. Kachru et al., *Towards inflation in string theory*, *JCAP* **10** (2003) 013 [hep-th/0308055] [SPIRES].

[180] D. Baumann et al., *On D3-brane potentials in compactifications with fluxes and wrapped D-branes*, *JHEP* **11** (2006) 031 [hep-th/0607050] [SPIRES].

[181] D. Baumann, A. Dymarsky, I.R. Klebanov, L. McAllister and P.J. Steinhardt, *A Delicate Universe*, *Phys. Rev. Lett.* **99** (2007) 141601 [arXiv:0705.3837] [SPIRES].

[182] A. Krause and E. Pajer, *Chasing Brane Inflation in String-Theory*, *JCAP* **07** (2008) 023 [arXiv:0705.4682] [SPIRES].

[183] D. Baumann, A. Dymarsky, I.R. Klebanov and L. McAllister, *Towards an Explicit Model of D-brane Inflation*, *JCAP* **01** (2008) 024 [arXiv:0706.0360] [SPIRES].

[184] E. Pajer, *Brane Inflation and Fine Tuning*, arXiv:0709.2239 [SPIRES].

[185] E. Pajer, *Inflation at the Tip*, *JCAP* **04** (2008) 031 [arXiv:0802.2916] [SPIRES].

[186] D. Baumann, A. Dymarsky, S. Kachru, I.R. Klebanov and L. McAllister, *Holographic Systematics of D-brane Inflation*, arXiv:0808.2811 [SPIRES].

[187] C. Herdeiro, S. Hirano and R. Kallosh, *String theory and hybrid inflation/acceleration*, *JHEP* **12** (2001) 027 [hep-th/0110271] [SPIRES].

[188] K. Dasgupta, C. Herdeiro, S. Hirano and R. Kallosh, *D3/D7 inflationary model and M-theory*, *Phys. Rev.* **D 65** (2002) 126002 [hep-th/0203019] [SPIRES].

[189] J.P. Hsu, R. Kallosh and S. Prokushkin, *On Brane Inflation With Volume Stabilization*, *JCAP* **12** (2003) 009 [hep-th/0311077] [SPIRES].

[190] C.P. Burgess, J.M. Cline, K. Dasgupta and H. Firouzjahi, *Uplifting and inflation with D3 branes*, *JHEP* **03** (2007) 027 [hep-th/0610320] [SPIRES].

[191] M. Haack et al., *Update of D3/D7-Brane Inflation on $K3 \times T^2/Z_2$*, *Nucl. Phys.* **B 806** (2009) 103 [arXiv:0804.3961] [SPIRES].

[192] C.P. Burgess, J.M. Cline and M. Postma, *Axionic D3-D7 Inflation*, arXiv:0811.1503 [SPIRES].

[193] R. Kallosh and A. Linde, *P-term, D-term and F-term inflation*, *JCAP* **10** (2003) 008 [hep-th/0306058] [SPIRES].

[194] P. Binetruy, G. Dvali, R. Kallosh and A. Van Proeyen, *Fayet-Iliopoulos terms in supergravity and cosmology*, *Class. Quant. Grav.* **21** (2004) 3137 [hep-th/0402046] [SPIRES].

[195] P.K. Tripathy and S.P. Trivedi, *Compactification with flux on K3 and tori*, *JHEP* **03** (2003) 028 [hep-th/0301139] [SPIRES].

[196] P.S. Aspinwall and R. Kallosh, *Fixing all moduli for M-theory on $K3 \times K3$*, *JHEP* **10** (2005) 001 [hep-th/0506014] [SPIRES].

[197] C. Angelantonj, R. D'Auria, S. Ferrara and M. Trigiante, *$K3 \times T^2/Z_2$ orientifolds with fluxes, open string moduli and critical points*, *Phys. Lett.* **B 583** (2004) 331 [hep-th/0312019] [SPIRES].

[198] R. D'Auria, S. Ferrara and M. Trigiante, *Orientifolds, brane coordinates and special geometry*, hep-th/0407138 [SPIRES].

[199] R. D'Auria, S. Ferrara and M. Trigiante, *No-scale supergravity from higher dimensions*, *Comptes Rendus Physique* **6** (2005) 199 [hep-th/0409184] [SPIRES].

[200] L. Görlich, S. Kachru, P.K. Tripathy and S.P. Trivedi, *Gaugino condensation and nonperturbative superpotentials in flux compactifications*, *JHEP* **12** (2004) 074 [`hep-th/0407130`] [SPIRES].

[201] J.F.G. Cascales and A.M. Uranga, *Branes on generalized calibrated submanifolds*, *JHEP* **11** (2004) 083 [`hep-th/0407132`] [SPIRES].

[202] O.J. Ganor, *A note on zeroes of superpotentials in F-theory*, *Nucl. Phys.* **B 499** (1997) 55 [`hep-th/9612077`] [SPIRES].

[203] L.J. Dixon, V. Kaplunovsky and J. Louis, *Moduli dependence of string loop corrections to gauge coupling constants*, *Nucl. Phys.* **B 355** (1991) 649 [SPIRES].

[204] C. Bachas and C. Fabre, *Threshold Effects in Open-String Theory*, *Nucl. Phys.* **B 476** (1996) 418 [`hep-th/9605028`] [SPIRES].

[205] I. Antoniadis, C. Bachas and E. Dudas, *Gauge couplings in four-dimensional type-I string orbifolds*, *Nucl. Phys.* **B 560** (1999) 93 [`hep-th/9906039`] [SPIRES].

[206] D. Lüst and S. Stieberger, *Gauge threshold corrections in intersecting brane world models*, *Fortsch. Phys.* **55** (2007) 427 [`hep-th/0302221`] [SPIRES].

[207] M. Berg, M. Haack and B. Körs, *Loop corrections to volume moduli and inflation in string theory*, *Phys. Rev.* **D 71** (2005) 026005 [`hep-th/0404087`] [SPIRES].

[208] M. Berg, M. Haack and B. Körs, *On the moduli dependence of nonperturbative superpotentials in brane inflation*, `hep-th/0409282` [SPIRES].

[209] N. Akerblom, R. Blumenhagen, D. Lüst and M. Schmidt-Sommerfeld, *Thresholds for intersecting D-branes revisited*, *Phys. Lett.* **B 652** (2007) 53 [`arXiv:0705.2150`] [SPIRES].

[210] N. Akerblom, R. Blumenhagen, D. Lüst and M. Schmidt-Sommerfeld, *Instantons and Holomorphic Couplings in Intersecting D-brane Models*, *JHEP* **08** (2007) 044 [`arXiv:0705.2366`] [SPIRES].

[211] C.L. Reichardt et al., *High resolution CMB power spectrum from the complete ACBAR data set*, `arXiv:0801.1491` [SPIRES].

[212] R. Bousso and S. Leichenauer, *Star Formation in the Multiverse*, `arXiv:0810.3044` [SPIRES].